1. A particle mass m is projected vertically upward in a constant gravitational field with an initial speed υ_o. If there is a frictional force proportional to the square of the instantaneous speed, $f = -\gamma\upsilon^2$,

(a) Write down the equation of motion for this particle. (5%)

(b) Find the terminal speed υ_t. (7%)

(c) Show that the speed of the particle when it returns to the initial position is $\upsilon_o\upsilon_t/\sqrt{\upsilon_o^2+\upsilon_t^2}$, where υ_t is the terminal speed. (13%)

2. An electrical circuit consists of a resistor R and a capacitor C connected in series to a source of alternating emf, $\varepsilon(t) = A\cos(\omega t)$.

(a) Write an equation for the instantaneous charge $Q(t)$. (6%)

(b) Make a comparison with the driving damped oscillation. (6%)

(c) Find the particular solution for the current as a function of time $I(t)$. (13%)

3. (a) Write down the expression of kinetic energy in spherical coordinator for a particle of mass m moving in a plane described by parameters (r,θ). (5%)

(b) Using the laws of energy conservation and angular momentum conservation, show that for a particle moving under a central potential $U(r)$ the radial speed equal to $\frac{dr}{dt} = \pm\sqrt{\frac{2}{m}(E-U)-\frac{L^2}{m^2r^2}}$, where the E is the total energy and L the angular momentum. (8%)

(c) Give $U(r) = -k/r$. At a specific time, the particle is found at a distance R from the center and moving with speed υ_o along a direction that makes an angle $30°$ with the radial vector. Find the maximum and minimum radial distances for this motion. (12%)

4. A particle of mass m moves in one dimension under the influence of a time-dependent force $f(x,t) = \frac{k}{x^2}\exp(-\frac{t}{\tau})$, where k and τ are positive constants.

(a) Find the Lagrangian function of this system. (8%)

(b) Compute the Hamiltonian function of this system. (8%)

(c) Discuss the conservation of energy for this system. (9%)

7205