1. (a) Prove that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force field. (b) Find the corresponding potential energy. (c) Find the work done in moving an object in this field from (1, -2, 1) to (3, 1, 4). (10%)

2. Prove that a bead P which is placed anywhere on a vertical frictionless wire, see Figure 1, in the form of a cycloid $x = b(\theta + \sin\theta)$, $y = b(1 - \cos\theta)$ will reach the bottom in the same time regardless of the starting point and find the time. (15%)

3. For a particle with mass m moving along a closed orbit in a central force field, (a) please show that its angular momentum is a conserving quantity, (b) show Kepler's second law is a direct consequence of part(a). (10%)

4. Construct a phase diagram for the potential $U(x) = -\frac{\lambda}{3}x^3$. (10%)

5. A particle moves in a plane under the influence of a force $f = -Ar^{\alpha-1}$ directed toward the origin; A and α (>0) are constants. Choose appropriate generalized coordinates, and let the potential energy be zero at the origin. (a) Find the Langrangian function, (b) Prove that the angular momentum about the origin is a conserved quantity. (c) Prove that the total energy is a conserved quantity. (20%)

6. Find the horizontal deflection from the plumb line caused by the Coriolis force acting on a particle falling freely in the Earth's gravitational field from a height h above the Earth's surface. (15%)

7. Two masses are connected by springs shown in figure 2. (a) Find the characteristic frequencies of the system. (b) Find the normal coordintes of the system. (20%)

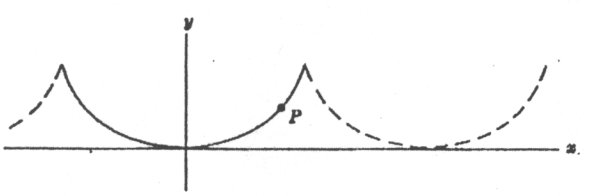

Figure 1

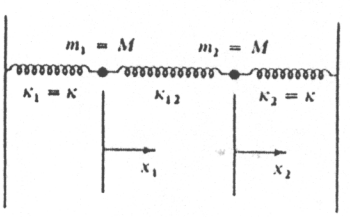

Figure 2