（每題20分，共100分）

1. A particle of mass $m$ is at rest at the end of a spring with constant $k$ hanging from a fixed support. At $t = 0$, a constant downward force $F$ is applied to the mass for a time $t_0$ and then removed. Write and solve the eqs. of motion for the time intervals $0 \le t < t_0$ and $t > t_0$ to show that, after the force is removed ($t > t_0$), the displacement from its equilibrium position is $x(t) = \frac{F}{k}\left[\cos\omega(t - t_0) - \cos\omega t\right]$, where $\omega^2 \equiv k/m$.

2. As shown in fig. 2, a solid cube of uniform density $\rho$ and sides of length $b$ is in equilibrium on top of a cylinder of radius $R$. Four sides of the cube are parallel to the axis of the cylinder. The contact between cube and cylinder is perfectly rough (no slipping). The cube is slightly displaced such that the angular position of the contact point (not the CM of cube) is $\theta$ from the equilibrium point. Use $M = \rho b^3$ & $I = Mb^2/6$ for shorter notations. (a) Find the gravitational potential energy $U(\theta)$ and the kinetic energy $T(\theta, \dot{\theta})$ (rotational plus translational which can be found by $v_{CM}^2 = \dot{x}^2 + \dot{y}^2$). (b) Write down the equation of motion for $\theta$, and find the frequency of small-amplitude oscillation ($R > b/2$ assumed).

3. We have a central force problem in which the angular momentum $\ell$, the total energy $E$, and the potential energy $U(r)$ are all known. (a) In terms of $E, \ell, U(r)$ and integrals involving them between appropriate limits ($r_{\min}, r_{\max}$ etc.), the angular velocity $\dot{\theta} =$ ? Radial velocity $\dot{r} =$ ? $\theta(r) =$ ? For $E < 0$, the apsidal angle $\Delta\theta =$? For $E > 0$ the scattering angle $\Psi =$ ? (b) If $U(r) = -\frac{k}{r}$ and the orbit is an ellipse given by $\frac{\alpha}{r} = 1 + \varepsilon\cos\theta$, show that $\alpha \equiv \frac{\ell^2}{mk}$ and $\varepsilon = \sqrt{1 + (2E\ell^2/mk^2)}$ by considering the energy and angular momentum at the pericenter (where $\theta \equiv 0$).

4. On the surface of Earth at a northern latitude $\lambda$, we set up a coordinate with the x-, y- and z-axes in the southerly, easterly and vertical directions respectively. A projectile is fired from the origin with initial velocity $(v_x, v_y, v_z)$. The Earth rotates at an angular velocity $\vec{\omega}$. By considering the Coriolis force ($\vec{a}_{co} = 2\vec{v}_r \times \vec{\omega}$) and using the total flying time $T \approx 2v_z/g$, find the position $(x, y, z)$ to order $O(\omega)$ where the projectile will strike the earth.

5. Show that the kinetic energy and the angular momentum of a rigid body with one point fixed are related to its angular velocity $\omega_i$ about that point by $T = \frac{1}{2}\sum_{i,j} I_{ij}\omega_i\omega_j$ and $L_i = \sum_j I_{ij}\omega_j$, where $I_{ij} \equiv \sum_\alpha m_\alpha\left(-x_{\alpha i}x_{\alpha j} + \delta_{ij}r_\alpha^2\right)$ with $r_\alpha^2 = \sum_k x_{\alpha k}^2$ is the inertia tensor.

fig.②