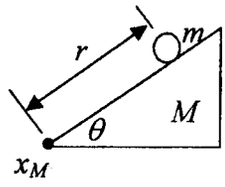
編號： 48　系所：物理學系　　科目：古典力學

本試題是否可以使用計算機： □可使用 ， ☑不可使用 （請命題老師勾選）

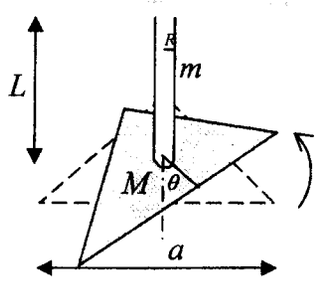
1. A cylinder of radius R and mass m goes down on a wedge of angle θ and mass M that can move without friction on a smooth horizontal surface as shown in Fig.1. Find the equations of motion (i.e. $r(t)$ and $x_M(t)$) for the cylinder and wedge when the cylinder is (a)(15%) slipping without rolling (b) (10%) rolling without slipping down the incline.

Fig. 1

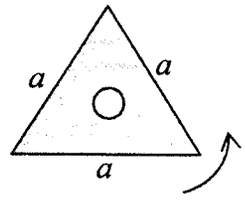


2. An equilateral triangle of mass M and side length a is suspended by a wire of mass m, length L and radius R from the ceiling. The central axis of the wire passes through the center of mass of the triangle. (a) (10%) Find the moment of inertia of this equilateral triangle. (b) (10%) Prove that $\tau=-\kappa\theta$, $\kappa = \pi R^4 S/2L$ where S is the shear modulus of this wire. (c) (5%) The oscillation frequency.

Fig.2



Top view



3. (25%) Prove that the Hamiltonian of a particle of mass m under the electromagnetic field is $H = |\mathbf{p} - q\mathbf{A}|^2/2m + q\phi$. If this particle is under the static electromagnetic field, we can get $\frac{1}{2}mv^2 + q\phi = E = \text{constant}$ from $\int \mathbf{F}\cdot d\mathbf{r} = -q\Delta\phi = \Delta T$. Is it a conflict between H and E?

4. (25%) An object in equilibrium consists of three balls of mass m connected by springs of length b and spring constant k. as shown in Fig.4. At time t=0, a small ball of mass m/2 and speed v collides with this object. If the collision time is very short and all the motions are just in one dimension, find the equations of motion of this object. (i.e. $x_1(t)$,$x_2(t)$,$x_3(t)$)

Fig. 4