

- (15%) A positron of energy $2m_e c^2$ annihilates with an electron of energy $m_e c^2$ (i.e. at rest) into two photons. (a) What was the velocity of the positron? (b) What is the maximum energy a photon can have?
- (10%) In Rutherford scattering, the scattering angle θ and the impact parameter b are related by $\cot(\theta/2) = 2b/D$, where D is some constant depending on the energy of particle. Show that the differential cross section is $d\sigma = \frac{D^2}{16} \frac{d\Omega}{\sin^4(\theta/2)}$, where $d\Omega$ is the solid angle.
- (10%) (a) Show that XP (or PX) can not be Hermitian. (b) We can have matrices of finite dimension to represent the algebra of angular momentum. But for the algebra $[X, P] = i\hbar$, show that we can not have finite-dimensional matrix representation.
- (15%) We have an delta function potential $V(x) = q\delta(x)$ in 1-dimension. (a) Show that the first derivative of wave function is discontinuous $\psi'(0+) - \psi'(0-) = (2mq/\hbar^2)\psi(0)$; (b) By assuming an incident wave e^{ikx} , a reflected wave Ae^{-ikx} and a transmitted wave Be^{ikx} , show that the transmission coefficient is $1/\left(1 + \frac{m^2 q^2}{k^2 \hbar^4}\right)$.
- (10%) (a) Show that the eigenstates of spin in $\hat{n} = (\cos\phi, \sin\phi, 0)$ direction $S_n = \vec{S} \cdot \hat{n}$ are $u_{\pm} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm e^{i\phi} \end{pmatrix}$. (b) If a measurement of S_x has yielded $+\hbar/2$, then a measurement of S_n is performed, what is the probability of yielding $+\hbar/2$ again?
- (15%) Two angular momenta $J_1 = 1\hbar$ & $J_2 = \hbar/2$ are to be added up $\vec{J}_t = \vec{J}_1 + \vec{J}_2$. Use the general relation $J_{\pm}|j, m\rangle = \sqrt{j(j+1) - m(m\pm 1)}|j, m\pm 1\rangle$ to find all $m_t \geq 0$ states by starting from $|3/2, 3/2\rangle = |1, 1\rangle|1/2, 1/2\rangle$ and using orthogonality between states.
- (15%) (a) Write down the complete singlet and triplet wave functions for 2-electron system occupying 2 space states $\psi_a(\vec{r})$ & $\psi_b(\vec{r})$ and spin states u or d (write particle 1 before particle 2 to avoid confusion). (b) Write down the energy shift due to the Coulomb interaction $\frac{e^2}{4\pi\epsilon_0 |\vec{r}_1 - \vec{r}_2|}$ in the form $\Delta E = J \pm K$ (you need to write out the details of J & K) and argue on physical ground that the energy is lower when the electron spins are parallel. (This is the origin of ferromagnetism.)
- (10%) 簡答題（一個名詞或一句話，顯示你知道就好）：
 - By what law do we know that the spectrum of any black body radiation must be identical to that of a rectangular metal cavity at same temperature?
 - It was found recently that the expansion of universe is accelerating, what is the possible source of this repulsive driving force?
 - Write down the 12 most elementary fermions known today that constitute all matters.
 - The Coulomb force between 2 electrons is repulsive, what extra force is responsible for forming 2-electron bound state (Cooper pair) in low temperature superconductor?
 - Explain the term "population inversion" in laser optics.