1. (a) From Maxwell's equations, derive the general electromagnetic boundary conditions between two media for $\vec{E}$, $\vec{D}$, $\vec{B}$, and $\vec{H}$. (Do **not** just write down them.)(8%) (b) From the results of (a), discuss the behaviors of electromagnetic fields across an interface between a perfect conductor and a perfect dielectric. (6%)

2. A positive point charge q is at the center of a spherical dielectric shell of an inner radius a and an outer radius b. The dielectric constant of the shell is ε_r. Find and plot (a) the electric field $\vec{E}$, (b) the electric potential V, and (c) the polarization $\vec{P}$ as functions of distance r to the center of the sphere. (15 %)

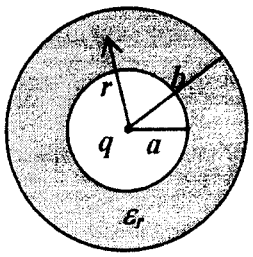

Fig. 1

3. A infinitely long, thin conducting cylindrical shell of radius a is split in two halves. The upper half is kept at a potential $+V_0$, and the lower half is kept at $-V_0$. Find the potential distribution both inside and outside the shell. (15%)
In cylindrical coordinates: $\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial V}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$.

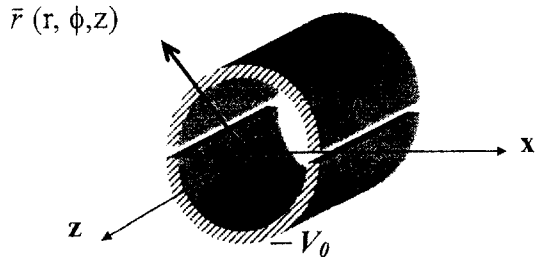

Fig. 2

(背面仍有題目,請繼續作答)

4. A parallel-plate capacitor is of width W, Length L, and separation d. A linear dielectric medium of dielectric constant ε_r is partially inserted into the capacitor. A fixed potential difference V_0 is kept between the plates. (a) Find $\vec{E}$, $\vec{D}$ (inside the capacitor) and surface charge density σ_s in the two separated regions. (6%) (b) Find the electric force on the dielectric. (5%) (c) Find the distance x such that the electrostatic energy stored in each region is the same. (5%)

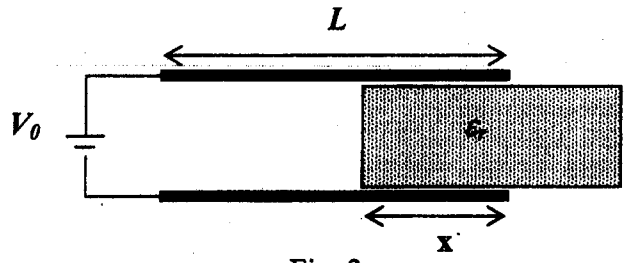

Fig. 3

5. Two coils of N_1 and N_2 turns are wound concentrically on a straight cylindrical core of radius a and permeability μ. The windings have length ℓ_1 and ℓ_2, respectively. (a) Find the self-inductances of the coils. (5%) (b) Find the mutual inductance between the coils. (5%) (c) If coil N_1 and N_2 carry currents I_1 and I_2, respectively, find the current ratio I_1/I_2 that makes the stored magnetic energy a minimum. (5%)

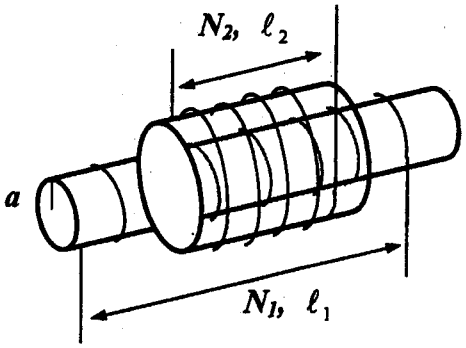

Fig. 4

6. (a) A small rectangular loop of side lengths a and b carries a current I. Find the vector potential $\vec{A}$ and magnetic induction $\vec{B}$ at an arbitrary point P, which is far away from the loop. (8%) (b) If a uniform magnetic field $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ is applied, determine the force and torque on the loop. (7%)

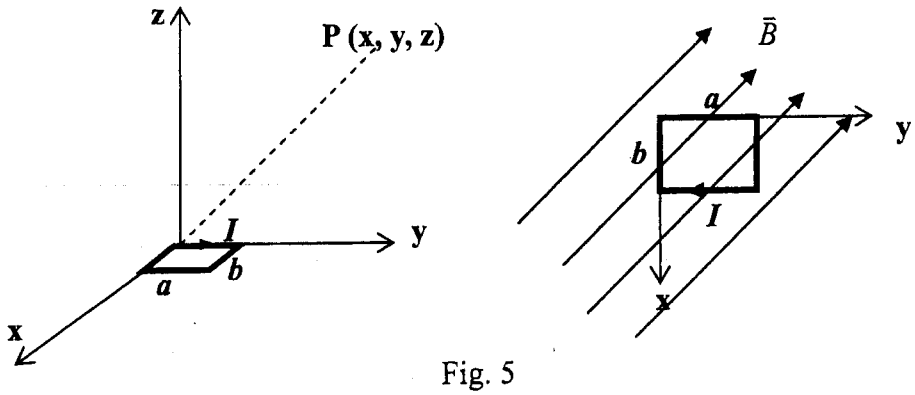

Fig. 5

7. A harmonic time-varying current density, $\vec{J}(\vec{r},t) = \vec{J}(\vec{r})e^{-i\omega t}$, is distributed in the free space. From the Maxwell's equations, show that if the Lorentz gauge is used, the magnetic-field can be determined by a vector potential $\vec{A}(\vec{r})$, which satisfies the Helmholtz equation:

$$\nabla^2\vec{A}(\vec{r}) + k^2\vec{A}(\vec{r}) = -\mu_0\vec{J}(\vec{r})$$

where $k^2 = \omega^2\mu_0\varepsilon_0$. Explain the physical meaning of k. (10%)