

※ 考生請注意：本試題 可 不可 使用計算機

1. (25 %) A particle of mass m is constrained to move on the inside surface of a smooth cone of half-angle α (see Fig. 1). The particle is subject to a gravitational force.
- (a) Find the Lagrange's equations of motion. (12 %)
- (b) If the particle moves in a circle of radius r_0 , what is the frequency, ω , of this motion? (3 %)
- (c) If the particle is then perturbed slightly from this circular motion, what is the frequency, Ω , of the oscillations about the radius r_0 ? (7 %)
- (d) Under what conditions does $\Omega = \omega$? (3 %)

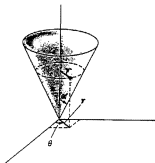


Fig. 1

2. (15 %) A particle moves in a potential, $U(r) = -C/(3r^3)$.
- (a) Given angular momentum l , find the maximum value of the effective potential. (5 %)
- (b) Let the particle come in from infinity with speed v_0 , what is the largest value of b (call it b_{max}) for which the particle is captured by the potential? In other words, what is the "cross section" for capture, πb_{max}^2 , for this potential? (10 %)
3. (25 %) Consider a particle moving in a potential $U(r)$. Rewrite the Lagrangian in terms of a coordinate system in uniform rotation with respect to an inertial frame.
- (a) Calculate the Hamiltonian and determine whether $H = E$. (10 %)
- (b) Is H a constant of the motion? Why? (3 %)
- (c) If E is not a constant of motion, why isn't it? (4 %)
- (d) Use the Lagrangian you obtained to reproduce the equations of motion (8 %)

$$\mathbf{F}_{eff} = \mathbf{F} - m\vec{\omega} \times (\vec{\omega} \times \vec{r}) - 2m\vec{\omega} \times \vec{v}$$

(背面仍有題目,請繼續作答)

系所組別：物理學系

考試科目：古典力學

考試日期：0306，節次：1

※ 考生請注意：本試題 可 不可 使用計算機

- 4.(20 %) A double pendulum consists of two simple pendula, with one pendulum suspended from the bob of the other. Assume the two pendulua have equal lengths and have bobs of equal mass and both pendula are confined to move in the same plane.
- (a) Find Lagrange's equations of motion for the system. (10 %)
- (b) For small oscillations, find the normal modes and their frequencies. (10 %)
- 5.(15 %) A stick of mass m and length l spins with frequency ω around an axis, as shown in Fig. 2. The stick makes an angle θ with the axis and is kept in its motion by two strings that are perpendicular to the axis. What is the tension in the strings? (Ignore gravity.)

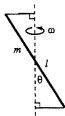


Fig. 2.