1. (15) The continuous function $f : \mathbb{R} \to \mathbb{R}$ has the property $|f(x) - f(y)| \geq |x - y|$, $\forall x, y \in \mathbb{R}$. Show that $f$ is surjective.

2. (15) A function $f$ is said to be Lipschitz continuous provided that there exists a constant $c$ such that $|f(x) - f(y)| \leq c|x - y|$, $\forall x, y$. Prove or disprove the following assertions:

   (a) (10) A differentiable function is Lipschitz continuous.

   (b) (5) A Lipschitz continuous function is differentiable.

3. (15) Show

   (a) (5)
   $$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = \sum_{n=0}^{\infty} \frac{1}{n!} \tag{1}$$

   (b) (10) the common value of equation (1) is irrational.

4. (10) What is the Jacobian of $f(x, y) = (\sin(\frac{x+y}{2}), 1 - \frac{x^2+y^2}{4})$ ?

5. (10) Find the volume of the ellipsoid $(x + 2y)^2 + (x - 2y + z)^2 + 3z^2 = 1$.

6. (10) Is it possible for a subset of real number to have an empty interior and the boundary of which is the whole set ? Justify your answer with a mathematical proof.

7. (15) A function is said to be conditionally integrable provided that it is improperly integrable but not absolutely integrable. Is the function $\sin x/x$ conditionally integrable on $[1, \infty)$ ?

8. (10) $D := \{(x, y) \in \mathbb{R} | \sqrt{x^2 + y^2} < \pi\}$. Define the function
   $$f(x, y) = \begin{cases} 0, & (x, y) = (0, 0) \\ \frac{x^2+y^2}{\sin(\sqrt{x^2+y^2})}, & (x, y) \in D \setminus \{(0, 0)\} \end{cases}$$
   Is $f$ differentiable at $(0, 0)$ ?