

系所組別： 數學系應用數學碩士班

考試科目： 高等微積分

考試日期：0223，節次：3

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Determine the limit of the following sequence and give an  $\varepsilon - N$  (definition) proof of convergence.

$$a_n = 1/\sqrt{2n+5}.$$

(20 points)

2. Let  $f$  be continuous on  $\mathbb{R}$  and that  $\lim_{x \rightarrow \pm\infty} f(x) = 0$ . Show that  $f$  will attain either a maximum on  $\mathbb{R}$  or a minimum on  $\mathbb{R}$  or both. (20 points)

3. Let  $f$  be differentiable on  $\mathbb{R}$  and that  $f'(x) \geq c > 0$  for all  $x$ . Show that for each  $y \in \mathbb{R}$ , there is a unique  $x$  such that  $f(x) = y$ . (20 points)

4. Define a function  $g$  by

$$g(x) = \begin{cases} 1, & x = 1/n, n \in \mathbb{N} \\ 0, & \text{otherwise.} \end{cases}$$

Show that  $\int_0^1 g = 0$ . (20 points)

5. Evaluate the double integral  $\int_0^2 \int_y^2 e^{x^2} dx dy$ . (20 points)