國立成功大學107學年度碩士班招生考試試題

系　　所：數學系應用數學

考試科目：線性代數

考試日期：0206，節次：1

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Remarks on notation. F is understood to be either $\mathbb{R}$ or $\mathbb{C}$. $F^{m\times n}$ denotes the class of all $m \times n$ matrices with entries in F.

Show all your work to get full credit.

1. $\beta = \{(4,-2),(6,-2)\}$ and $\gamma = \{(1,1),(1,-1)\}$ are two ordered bases of F^2. Find the change of basis matrix from β to γ. (15 points)

2. Let $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$. Derive the formula of A^n ($n \in \mathbb{N}$). (15 points)

3. The equation $13x^2 - 10xy + 13y^2 - 72 = 0$ determines an ellipse in the plane $\mathbb{R}^2$. Find its area. (15 points)

4. Apply the Gram-Schmidt process to the vectors $(1,0,1),(1,0,-1),(0,3,4)$, to obtain an orthonormal basis for $\mathbb{R}^3$ with the standard inner product. (16 points)

5. Let $A, B \in F^{n\times n}$. Prove that if $I - AB$ is invertible, then $I - BA$ is also invertible. (15 points)

6. Let $(V, \langle\cdot,\cdot\rangle)$ be a finite dimensional inner product space over F. For a subspace K of V, let $K^\perp$ denote its orthogonal complement $\{v \in V \mid \langle v, w\rangle = 0 \ \forall w \in K\}$. Let K_1, K_2 be two subspaces of V. Prove the following formulas.
   (a) $(K_1 + K_2)^\perp = K_1^\perp \cap K_2^\perp$. (12 points)
   (b) $(K_1 \cap K_2)^\perp = K_1^\perp + K_2^\perp$. (12 points)