國立成功大學109學年度碩士班招生考試試題

系　所：數學系應用數學

考試科目：高等微積分

考試日期：0211，節次：2

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) Show that a sequence of real numbers converges if and only if it is a Cauchy sequence.

2. (20%) Suppose that f is a continuous real function defined on $\mathbb{R}$ and $\mathcal{O}$ is an open subset of $\mathbb{R}$. Determine whether the following statements are true of false. Prove the statement if it is true, and give a counterexample if it is false.

   a. (10%) $f^{-1}(\mathcal{O})$ is open in $\mathbb{R}$.

   b. (10%) $f(\mathcal{O})$ is open in $\mathbb{R}$.

3. (30%) Suppose that $\{f_n\}$ is a sequence of real functions defined on $[0,1]$ and that $f_n \to f$ uniformly on $[0,1]$. Show that

   a. (15%) If f_n is continuous on $[0,1]$ for each $n \in \mathbb{N}$, then f is uniformly continuous on $[0,1]$.

   b. (15%) If f_n is Riemann integrable on $[0,1]$ for each $n \in \mathbb{N}$, then f is Riemann integrable on $[0,1]$ and

$$\int_0^1 f(x)dx = \lim_{n\to\infty} \int_0^1 f_n(x)dx.$$

4. (10%) Find the 2nd-order Taylor polynomial of $f(x,y) = \sqrt{x} + \sqrt{y}$ about $(x,y) = (1,4)$.

5. (20%) Determine whether the following statements are true of false. Prove the statement if it is true, and give a counterexample if it is false.

   a. (10%) If f is Riemann integrable on $[a,b]$, then the function $F(x) = \int_a^x f(t)dt$ is differentiable on (a,b) and $F'(x) = f(x)$ for $x \in (a,b)$.

   b. (10%) If $|f|$ is Riemann integrable on $[a,b]$, then f also is Riemann integrable on $[a,b]$.