

國立成功大學

110學年度碩士班招生考試試題

編 號：35

系 所：數學系應用數學

科 目：線性代數

日 期：0203

節 次：第 1 節

備 註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Let $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & -2 & 3 \\ 0 & -1 & 2 \\ 0 & 1 & 0 \end{bmatrix}$. Find an orthonormal basis (with respect to the standard inner product on \mathbb{R}^4) for the range of A . (15 points)

2. Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}.$$

Find the trace of the linear operator $X \mapsto AXB$ on $\mathbb{R}^{2 \times 2}$. (10 points)

3. Let $M = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \in \mathbb{C}^{2n \times 2n}$, where A, B, C are $n \times n$ matrices and 0 is the $n \times n$ zero matrix. Prove that M is invertible if and only if both B and C are invertible. (15 points)

4. Let $U \in \mathbb{C}^{n \times n}$ be a unitary matrix. Prove that there exists Hermitian matrix M such that $U = e^{iM}$. (15 points)

5. Let $P_2(\mathbb{R})$ be the real vector space of quadratic polynomials, and let $T : P_2(\mathbb{R}) \rightarrow P_2(\mathbb{R})$ be the linear transformation defined by

$$(Tf)(x) = f(x) + f'(x) + f''(x).$$

If β denotes a basis for $P_2(\mathbb{R})$, let us use $[T]_\beta$ to denote the matrix representation of T with respect to β .

- (a) Find $[T]_\beta$ with $\beta = (1, x, x^2)$. (10 points)
- (b) Is T diagonalizable? If yes, find the basis β for $P_2(\mathbb{R})$ such that $[T]_\beta$ is diagonal. If no, find the basis β such that $[T]_\beta$ is a Jordan form. (15 points)
6. (a) Give the 3×3 real matrix which is the rotation by an angle θ about the z -axis. (For this question you are allowed to give the answer directly without justification.) (5 points)
- (b) Find the 3×3 real matrix which is the rotation by the angle $\pi/2$ about the axis along $(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})$. (15 points)