國立成功大學
110學年度碩士班招生考試試題

編　號：36

系　所：數學系應用數學

科　目：高等微積分

日　期：0203

節　次：第 2 節

備　註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) Determine the following limit by definition.
$$\lim_{x \to 1}(x^3 - x^2 + 3x - 2)$$

2. (10%) Determine the following limit by definition.
$$\lim_{n \to \infty} \frac{n^2}{2^n}$$

3. (10%) Show that the following set is open by definition.
$$\{x^2 + 4y^2 < 4\} \subset \mathbb{R}^2$$

4. (15%) Evaluate the limit. Justify your answer.
$$\lim_{n \to \infty} \left(\int_0^1 x^n(1-x)^n dx \right)^{\frac{1}{n}}.$$

5. (15%) Define $f : \mathbb{R}^2 \to \mathbb{R}$ by $f(0,0) = 0$ and
$$f(x,y) = \frac{x^{\frac{4}{3}}y^{\frac{2}{3}}}{(x^2+y^2)^{\frac{1}{2}}}, \quad (x,y) \neq (0,0).$$

   (a) Is f continuous at $(0,0)$?

   (b) Is f differentiable at $(0,0)$?

6. (20%) Define function f on all possible $x \in \mathbb{R}$ such that the limit exists:
$$f(x) = \lim_{n \to \infty} \frac{x^{1+n}}{1+x^n}.$$

   (a) Determine the domain and the values of f explicitly. Sketch the graph of f.

   (b) Is the convergence uniform on $[1,2]$?

   (c) Is the convergence uniform on $[2,3]$?

7. (20%) Suppose $\{f_n\}_{n \in \mathbb{N}}$ is a sequence of real-valued continuous functions on $[0,1]$ which is uniformly convergent to f on $[0,1]$.

   (a) Show that f is continuous on $[0,1]$.

   (b) Show that the family $\{f_n\}_{n \in \mathbb{N}}$ is uniformly bounded on $[0,1]$.

   (c) Show that the family $\{f_n\}_{n \in \mathbb{N}}$ is equicontinuous on $[0,1]$.