國立成功大學
111學年度碩士班招生考試試題

編　號：35

系　所：數學系應用數學

科　目：線性代數

日　期：0220

節　次：第 1 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (12 points) Find the rank of the real matrix

$$\begin{bmatrix} 3 & 3 & -2 & -1 \\ 8 & 6 & -4 & 2 \\ -9 & -9 & 6 & 3 \\ 1 & 0 & 0 & 2 \end{bmatrix}$$

2. (10 points) Let A and B be 9×9 real matrices such that $AB = -BA$. Show that A or B is not invertible.

3. (16 points) Let $\mathbb{R}$ denote the field of all real numbers, and let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by

$$T(a,b) = (3a + 5b, a - 2b)$$

for all $(a,b) \in \mathbb{R}^2$. Let $\beta = \{v_1, v_2\}$, where $v_1 = (1,-2)$ and $v_2 = (3,-1)$. Describe the matrix of T with respect to the ordered basis β for $\mathbb{R}^2$.

4. Consider the real matrix

$$A = \begin{bmatrix} 2 & 0 & 14 \\ -7 & 16 & 7 \\ 0 & 0 & 16 \end{bmatrix}.$$

   (a) (10 points) Find all eigenvalues of A.

   (b) (16 points) Is it true that there exists a 3×3 real matrix B such that $B^4 = A$? *Justify your answer.*

5. Let V be the real vector space consisting of all $n \times n$ real matrices. For $A, B \in V$, define $\langle A, B \rangle$ to be the trace of AB^T. (Here B^T denotes the transpose of B.)

   (a) (10 points) Show that $\langle \cdot, \cdot \rangle$ is an inner product on V.

   (b) (10 points) Let $P \in V$ be a fixed invertible matrix, and let $T\colon V \to V$ be the linear transformation defined by $T(A) = P^{-1}AP$ for all $A \in V$. Describe the adjoint of T with respect to the inner product $\langle \cdot, \cdot \rangle$.

6. (16 points) Let F be any field, and let n be any positive integer. Suppose that A and B are $n \times n$ matrices over F such that A and B are diagonalizable and $AB = BA$. Show that there exists an invertible $n \times n$ matrix P over F such that both $P^{-1}AP$ and $P^{-1}BP$ are diagonal matrices.