5/6 (c)

國立成功大學七十六學年度研究所入學考試(基礎數學(B)試題) 共 3 頁 第 1 頁

注意事項:

(1) 本試題共分 PART I (Linear Algebra), PART II (probability) 及 PART III (Numerical Analysis) 三部分. 每部分 50分.

(2). 請選兩部分作答(請將題號標示清楚).

PART I (Linear Algebra)

1. Given $A = \begin{bmatrix} -1 & -1 & -3 & 1 & -1 \\ 2 & 1 & 7 & -4 & 1 \\ 1 & 0 & 4 & -3 & 1 \\ 3 & 1 & 11 & -7 & 2 \\ 2 & 0 & 8 & -6 & 1 \end{bmatrix}$

(a) Find an invertible matrix P such that PA is a row-reduced echelon matrix R. (10%)

(b) Find a basis for the row space W of A. (4%)

(c) Let V be the vector space of all 5×1 column matrices X such that $AX=0$. Find a basis for V. (8%)

2. (i) Let λ_1 and λ_2 be distinct eigenvalues of a matrix A, with associated eigenvectors X_1 and X_2 respectively. Show that X_1 and X_2 are linearly independent. (8%)

(ii) Let A be an $n\times n$ matrix such that $A-I$ is nilpotent (where I is the identity matrix). prove that $\det A = 1$. (8%)

3. Let T and S be linear operators from V to V, where V is a vector space of dimension $n\geq 2$, satisfying $TS=ST$, $T^2=0$, $T\neq 0$. Prove that S has a non-trivial invariant subspace. (12%)

Part II (Probability) (每題 10 分)

1. 設函數 $F:\mathbb{R}\rightarrow\mathbb{R}$ 而 $F(x)=\begin{cases}0, & \\ \frac{1}{\pi}\tan^{-1}x+\frac{1}{2}, & 若\ x\geq 0.\end{cases}$

   試問 F 是否為某隨機變數之累積分配函數(c.d.f.)? 為何?

2. 設隨機變數 X 具常態分配 $N(\mu,\sigma^2)$,試証:

   $$\forall k>0,\quad P\{|X-\mu|\geq k\sigma\}<\frac{1}{k^2}.$$

   (注意:是 <,不是 ≤).

3. 設隨機變數 X 具 Laplace 分配,即其機率密度函數(p.d.f.)為

   $$f(x)=c\,e^{-|x|},$$ 內 c 為一常數.

   試求 X 之特徵函數.

4. 設隨機向量 (X, Y) 為絕對連續型. 試問:陳述

   "X 與 Y 為不相關(uncorrelated) ⇒ X 與 Y 為獨立."

   是否為真? 若為真,詳証之;若不然,則舉一反例,並詳細說明之.

5. 若隨機變數序列 $\{X_n\}_n$ 機率收斂于某隨機變數 X (X_n converges in probability to X),証明:X 為殆必唯一 (a.s. unique). 亦即証明:若 $X_n \xrightarrow{P} X$ 且 $X_n \xrightarrow{P} Y$,則 $X=Y$ a.s.

   (提示:$|X-Y|\leq|X_n-X|+|X_n-Y|$.)

PART Ⅲ (數值分析)

1. Let $f:[-1,3]\to\mathbb{R}$ be such that $f(-1)=-1$, $f(0)=3$, $f(2)=1$ and $f(3)=27$. Use the

   (i) Lagrange form to find the quadratic interpolating polynomial $P(x)$, and (10%)

   (ii) Newton form to find the cubic interpolating polynomial $Q(x)$. (10%)

2. Let $f(x)=c_0+c_1x+c_2x^2+c_3x^3$. Show that

   (i) the corrected trapezoidal rule can be written as
   $$\int_{-h}^{h} f(x)\,dx = h\left[f\left(\frac{h}{\sqrt{3}}\right)+f\left(\frac{-h}{\sqrt{3}}\right)\right],$$ and (10%)

   (ii) the Simpson's rule gives the exact value of $\int_{-h}^{h} f(x)\,dx$ (5%)

3. Given the differential equation $y'=2x+y$, with $y(0)=1$, use the following methods to approximate $y(0.1)=?$

   (i) Euler's method with $h=0.05$. (5%)

   (ii) Runge-Kutta method of 4th order with $n=1$. (10%)