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1 Consider the model, for $i=1,2,\cdots,n$,

$$Y_j = \beta_1 + \beta_2 x_j + e_j,\ E(e_j)=0,\ E(e_i e_j) = \begin{cases} \sigma^2 & \text{if } i=j \\ 0 & \text{if } i\neq j \end{cases},\ \beta_1, \beta_2,$$

σ^2 are unknown, x_i's are known.

10% (a) Find the least square estimates of β_1 and β_2.

10% (b) Suppose that $x_i \neq x_j$ for some $1 \le i, j \le n$, Find the variances and convariance of β_1, β_2.

2 Let $X_1,\cdots,X_n$ be iid r.v's and $X_1 = 1, 2$ and 3 with probabilities θ^2, $2\theta(1-\theta)$ and $(1-\theta)^2$ respectively. Let n_1, n_2, n_3 denote the numbers of $\{X_1,\cdots,X_n\}$ equal to 1, 2 and 3.

10% (a) Find the joint probability distribution of $X_1,\cdots,X_n$.

10% (b) Find the maximum likehood estimate of θ.

3. Let $X_1,\cdots,X_n$ be iid according to the exponential distribution $E(a,b)$, $-\infty<a<\infty$, $0<b$, with density $f(x) = \frac{1}{b} e^{-(x-a)/b}$ for $x>a$. It's well known that $T_1 = X_{(1)}$, the smallest order statistic of $X_1,\cdots,X_n$ and $T_2 = \sum_{i=1}^{n}[X_i - X_{(1)}]$ are independently distributed as $E(a, b/n)$ and $\frac{1}{2}b\chi^2_{2n-2}$ respectively and they are jointly sufficient and complete.

10% (a) If b is known, find the U.M.V.U. estimate of a.

10% (b) If both a and b are unknown, find the U.M.V.U. estimate of a and b respectively.

4. Let $X_1, \cdots, X_n$ be a random sample from $B(1, \theta)$ binormial distribution, $\theta \in (0,1)$. Consider the problem of estimating θ with a square error loss function. If the prior distribution λ of θ is $\beta(\alpha, \beta)$; that is

$$\lambda(\theta) = \begin{cases} \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1} & \text{if } \theta \in (0,1). \\ 0 & \text{otherwise.} \end{cases}$$

10% (a) Find the corresponding Bayes estimate of θ.

10% (b) Find the minimax estimate of θ.

5.

5% (a) State the Neyman-Pearson lemma.

5% (b) State the definition of an uniformly most powerful (U.M.P) test for $H: \theta \in \omega$ vs $K: \theta \in \omega^c$.

10% (c) Suppose that $X_1, \cdots, X_n$ is a sample from $N(\mu, 1)$. Find the level α U.M.P test for $H: \mu \leq 0$ vs $K: \mu > 0$.