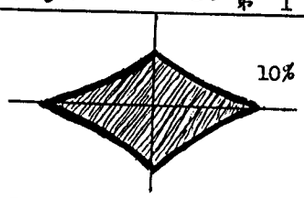
1. Find the area inclosed by the curve

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1 \quad (a,b > 0).$$



10%

2. f is twice differentiable and satisfies $|f(x)| < A$, $|f''(x)| < B$, for all real x, where A and B are constants. Prove that $|f'(x)| < 2\sqrt{AB}$. 15%

3. Given $\int_0^{\pi/2} \frac{dx}{\sin^2 x + y^2 \cos^2 x} = \frac{\pi}{2y}$ $(y > 0)$.

Find $G(y) = \int_0^{\pi/2} \ln(\sin^2 x + y^2 \cos^2 x)\, dx$.

[Suggestion. To find $(y^2 - 1)G'(y)$ first.] 20%

4. (a) Let $f(x) = e^x$,

$$g(x) = \begin{cases} 1 & \text{if } x = 0 \\ x & \text{if } 0 < x < 1 \\ 0 & \text{if } x = 1. \end{cases}$$

Evaluate $\int_0^1 f\, dg$. 10%

(b) If $f(x) \geq 0$ is bounded and measurable on set E and $\int_E f(x)\, dx = 0$, prove that $f(x) = 0$ almost everywhere on E. 15%

5. (a) Solve the following differential equations:

(i) $y'' + y = \tan x$;

(ii) $\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{-(x^2 + y^2)}$. 20%

(b) If y_1 and y_2 are two solutions of the differential equation

$$y'' + p_1(x)\, y' + p_2(x)\, y = 0$$

on an interval I where $p_1(x)$ and $p_2(x)$ are continuous, and if the wronskian of y_1 and y_2 vanishes anywhere (that is, at even one point) in I, prove that y_1 and y_2 are linearly dependent over I. 10%