1. Let $X_1, X_2, \cdots, X_n$ be a random sample from a population defined by the Cauchy density with location parameter θ:

$$f(x;\theta) = \frac{1}{\pi[1+(x-\theta)^2]}, \quad -\infty < x < \infty.$$

Find the minimal sufficient statistic of θ.

2. Let $X_1, X_2, \cdots, X_n$ be a random sample from a population defined by the negative exponential distribution with parameter $\theta \in (0, \infty)$.

$$f(x;\theta) = \theta e^{-\theta x}, \quad x > 0.$$

(a) Find the U.M.V.U. estimator of the reliability $R(x;\theta)$, where

$$R(x;\theta) = P_\theta\{X_1 > x\}.$$

(b) Find the M.L.E. of $R(x;\theta)$.

3. On the basis of the following score, approximately taken, test whether there are sex associated differences in mathematical ability. Take $\alpha = 0.05$

Boys :	80	96	98	87	75	83	70	92	97	82
Girls :	82	90	84	70	80	97	76	90	88	86

4. Let X be Poisson distributed with parameter $\theta > 0$, and $L(\theta, a) = (\theta - a)^2$ be the loss function. $f(x;\theta) = e^{-\theta}\theta^x/x!$ $x = 0, 1, 2, \cdots$

Take as the prior distribution of θ the gamma distribution $G(\alpha, \beta)$ with density, where $\alpha > 0$, $\beta > 0$.

$$g(\theta) = (\Gamma(\alpha)\beta^\alpha)^{-1} e^{-\theta/\beta}\theta^{\alpha-1} \quad \text{for } \theta > 0$$

(a) Find the posterior distribution of θ given $X = x$.

(b) Find the Bayes estimate with respect to $G(\alpha, \beta)$. (Hint: the first two moments of $G(\alpha, \beta)$ are $\alpha\beta$ and $\alpha(\alpha+1)\beta^2$)

5. Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(0, \theta)$ with density

$$f(x;\theta) = (2\pi\theta)^{-\frac{1}{2}} e^{-x^2/2\theta}, \quad \theta > 0$$

(a) Show that $T = \sum_{i=1}^{n} X_i^2$ is a sufficient statistic for θ

(b) Find the information $I_T(\theta)$ provided by T.

Critical Values for Student's t-Distribution

Let t_r be a random variable having the Student's t-distribution with r degrees of freedom. Then the tabulated quantities are the numbers x for which

$$P(t_r \leq x) = \gamma.$$

	γ					
r	0.75	0.90	0.95	0.975	0.99	0.995
1	1.0000	3.0777	6.3138	12.7062	31.8207	63.6574
2	0.8165	1.8856	2.9200	4.3027	6.9646	9.9248
3	0.7649	1.6377	2.3534	3.1824	4.5407	5.8409
4	0.7407	1.5332	2.1318	2.7764	3.7649	4.6041
5	0.7267	1.4759	2.0150	2.5706	3.3649	4.0322
6	0.7176	1.4398	1.9432	2.4469	3.1427	3.7074
7	0.7111	1.4149	1.8946	2.3646	2.9980	3.4995
8	0.7064	1.3968	1.8595	3.3060	2.8965	3.3554
9	0.7027	1.3830	1.8331	2.2622	2.8214	3.2498
0	0.6998	1.3722	1.8125	2.2281	2.7638	1.1693
1	0.6974	1.3634	1.7959	2.2010	2.7181	3.1058
2	0.6955	1.3562	1.7823	2.1788	2.6810	3.0545
3	0.6938	1.3502	1.7709	1.1604	2.6503	3.0123
4	0.6924	1.3450	1.7613	2.1448	2.6245	2.9768
5	0.6912	1.3406	1.7531	2.1315	2.6025	2.9467
6	0.6901	1.3368	1.7459	2.1199	2.5835	2.9208
7	0.6892	1.3334	1.7396	2.1098	2.5669	2.8982
8	0.6884	1.3304	1.7341	2.1009	2.5524	2.8784
9	0.6876	1.3277	1.7291	2.0930	2.5395	2.8609
0	0.6870	1.3253	1.7247	2.0860	2.5280	2.8453
1	0.6864	1.3232	1.7207	2.0796	2.5177	2.8314
2	0.6858	1.3212	1.7171	2.0739	2.5083	2.8188
3	0.6853	1.3195	1.7139	2.0687	2.4999	2.8073
4	0.6848	1.3178	1.7109	2.0639	2.4922	2.7969
5	0.6844	1.3163	1.7081	2.0595	2.4851	2.7874
6	0.6840	1.3150	1.7056	2.0555	2.4786	2.7787
7	0.6837	1.3137	1.7033	2.0518	2.4727	2.7707
8	0.6834	1.3125	1.7011	2.0484	2.4671	2.7633
9	0.6830	1.3114	1.6991	2.4052	2.4620	2.7564
0	0.6828	1.3104	1.6973	2.0423	2.4573	2.7500
1	0.6825	1.3095	1.6955	2.0395	2.4528	2.7440
2	0.6822	1.3086	1.6939	2.0369	2.4487	2.7385
3	0.6820	1.3077	1.6924	2.0345	2.4448	2.7333
4	0.6818	1.3070	1.6909	2.0322	2.4411	2.7284
5	0.6816	1.3062	1.6896	2.0301	2.4377	2.7238