國立成功大學 79 學年度應用數學研究所入學考試(基礎數學(B)試題) 共 1 頁 第 1 頁

Part II (Probability)

1. Let $\{E_i ; i \geq 1\}$ be a sequence of events of a sample space S. Suppose that $Pr(E_i) = 1$ for all $i \geq 1$. Prove or disprove that $Pr(\bigcap_1^{\infty} E_i) = 1$. (10%)

2. Let $Z_1, Z_2, \ldots$ be a sequence of random variables and suppose that for n = 1, 2, ... , the distribution of Z_n is as follows:

$$Pr(Z_n = \frac{1}{n}) = 1 - \frac{1}{n^2} \quad \text{and} \quad Pr(Z_n = n) = \frac{1}{n^2}.$$

Does there exist a constant c to which the sequence converges in probability? (10%)

3. Suppose that by any time t the number of people that have arrived at a train depot is a Poisson random variable with mean λt. If the initial train arrives at the depot at a time (independence of when the passengers arrive) that is uniformly distributed over (0, T), what is the mean and variance of the number of passengers that enter the train? (10%)

4. Consider the function $F(x_1, x_2)$ defined as follows:

$$F(x_1, x_2) = \begin{cases} 0.25(x_1 + x_2)^2 & \text{if } 0 \leq x_1 < 1 \text{ and } 0 \leq x_2 < 1 \\ 0 & \text{if } x_1 < 0 \text{ or } x_2 < 0 \\ 1 & \text{otherwise.} \end{cases}$$

Is $F(x_1, x_2)$ a bivariate CDF? Explain your answer. (10%)

5. Let X and Y be two independent random variables. Suppose that X and Y - X are independent. Show that X has a degenerate distribution at some constasnt c. (10%)