

Numerical Analysis (數值分析) 50%

I. Find a straight line in the plane that comes closest to fitting the points:

$$(0, 1), (1, 1.9), (2, 3.1), (3, 3.9).$$

relative to $\| \cdot \|_2$.

(10%)

II. Let $f(x) = \frac{1}{x+1}$. Find a polynomial $P_3(x)$ of degree ≤ 3 such that

(a) $P_3(i) = f(i)$, for $i = 0, 1, 2, 3$. (by Newton's forward formula).

(10%)

(b) $P_3(i) = f(i)$ and $P_3'(i) = f'(i)$, for $i = 0, 1$.

(10%)

III. Consider the initial value problem

$$y' = 2xy, \quad y(0) = 1, \quad 0 \leq x \leq 1.$$

(a) Derive the third-order Taylor's series method

$$y_{i+1} = y_i + h T_3(x_i, y_i, h).$$

(Write out $T_3(x_i, y_i, h) = ?$).

(10%)

(b) Choose the maximal step size h (equally spaced) such that the local truncation error $\tau_i < 0.005$.

(10%).