(12%) 1. Let H be a subgroup of the group G and let N be a normal subgroup of G. Show that $\dfrac{HN}{N} \cong \dfrac{H}{H \cap N}$.

(12%) 2. Let G be a group of order 35. Show that G is an abelian group.

(12%) 3. Let R be a commutative ring with unity. Show that R is a field if and only if R has no proper nontrivial ideals.

(14%) 4. Let ζ be a primitive 5-th root of unity in $\overline{Q}$ (the algebraic closure of the rational field Q).

(1) Find the Galois group of $Q(\zeta)$ over Q. (6%).

(2) Find all the fields between $Q(\zeta)$ and Q. (8%).

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