國立成功大學七十九學年度應用數學研究所考試(常微分方程 試題) 共 1 頁 第 1 頁

1. (A) Use two different methods to solve $y'' = f(x)$, $y(0) = y'(0) = 0$, where f is a continuous function. 10%

   (B) Determine the interval of x for which the solution of the following equation is defined :

$$x^2(1 + y^2)\,dx + 2y\,dy = 0, \quad y(0) = 1.$$ 10%

2. Solve the following three equations:

   (A) $\frac{d^{n+1}y}{dx^{n+1}} - 2\frac{d^n y}{dx^n} = 5$, where n is a positive integer; 10%

   (B) $xy'' + 2y' - y = 0$; 10%

   (C) $y'' - y = 2\delta(x-2)$, $y(0) = y'(0) = 0$, where δ is the Dirac δ–function. 10%

3. (A) Let u and v be real linearly independent solutions of $a(x)y'' + b(x)y' + c(x)y = 0$, where a, b, and c are real functions. Show that $y_1 = u + iv$ and $\overline{y_1} = u - iv$ (i.e. $\overline{y_1}$ is the complex conjugate of y_1) are complex solutions. Also show that the general complex solution is $y = c_1 y_1 + c_2\overline{y_1}$, where c_1 and c_2 are arbitrary complex numbers. 10%

   (B) Show that $y = x^{1+i}$ satisfies $x^2y'' - xy' + 2y = 0$. What are real solutions of the equation ? 10%

4. (A) Let A be an n by n constant matrix. Show that the general solution of the differential equation $\frac{d\vec{x}(t)}{dt} = A\vec{x}(t)$ is given by $\vec{x}(t) = (\exp(tA))\vec{c}$, where $\vec{c}$ is an arbitrary constant vector. The unique solution of the differential equation which also satisfies the initial condition $\vec{x}(t_0) = \vec{x}_0$ is given by $\vec{x}(t) = (\exp((t - t_0)A))\vec{x}_0$. 10%

   (B) Solve $\frac{d\vec{x}}{dt} = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}\vec{x}$, $\vec{x}(0) = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$. 10%

5. Find the steady–state solution to the equation $\ddot{x} + 2\dot{x} + 2x = 2\cos(3t)$. Also estimate the earliest time beyond which the transient solution remains less than 0.01, if we are given the initial conditions $x(0) = 1$, $\dot{x}(0) = 0$. 10%

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