(甲)

國立成功大學七十九學年度應數所考試(數理統計 試題) 共 2 頁 第 1 頁

10% 1. Let $X_1, \cdots, X_n$ be a random sample from a distribution with pdf $f(x;\theta)$. Let $T(X_1, \cdots, X_n)$ be a complete sufficient statistic for the family $\{f(x;\theta)\}$. If $U(X_1, \cdots, X_n)$ is a statistic (not a function of T alone) whose distribution does not depend on θ, show that U is independent of T.

20% 2(a) Let X be a random variable with pdf $f(x;\theta) = \theta e^{-\theta x}$, $x > 0$, $\theta > 0$. Find the UMVU estimator of the reliability $R(x;\theta) = P_\theta\{X > x\}$ on the basis of n observations on X.

(b) If $X_1, \cdots, X_n$ are i.i.d. r.v.'s with pdf $f(x;\theta) = \theta e^{-\theta x}$, $x > 0$, $\theta > 0$. find the MLE of θ

20% 3(a) Let $\{Z_n\}$ be a sequence of r.v.'s. What is a stopping time defined on $\{Z_n\}$?

(b) Let Z_n be independent r.v's with identically first moment such that $E|Z_j| = M < \infty$ so that $EZ_j = \mu$ is also finite. Let N be a stopping time and EN is finite. Show that $ES_N = \mu EN$ where $S_N = Z_1 + \cdots + Z_N$.

20% 4. 下表為10位推銷員之智力測驗分數及每週銷售額:

銷售員	1	2	3	4	5	6	7	8	9	10	
智力測驗 (x)	50	70	60	80	70	90	60	50	70	60	
每週銷售額 (y)	3.5	6.0	4.0	5.5	5.0	7.0	3.0	3.0	4.5	5.0	(單位:千元)

(a) 求銷售額y對智力測驗之迴歸直線方程式.

(b) 在 $x = 75$ 情況下, 若 $\alpha = 0.05$; 求相對之每週銷售額之區間估計.

附: $\sum_{i=1}^{10} x_i = 660$, $\sum_{i=1}^{10} y_i = 46.5$, $\sum_{i=1}^{10} x_i y_i = 3200$, $\sum_{i=1}^{10} x_i^2 = 45000$, $\sum_{i=1}^{10} y_i^2 = 231.75$

$t_{8,0.025} = 3.3060$ $t_{9,0.025} = 2.2622$, $t_{8,0.05} = 1.8595$, $t_{9,0.05} = 1.8331$

036

(甲)

10% 5. Let $X_1, \cdots, X_m$ and $Y_1, \cdots, Y_n$ be two independent random sample with pdf's f_1 and f_2, respectively,

$$f_1(x;\theta_1) = \frac{1}{\theta_1} e^{-\frac{x}{\theta_1}} \quad , x>0 \ , \theta_1 > 0.$$

$$f_2(y;\theta_2) = \frac{1}{\theta_2} e^{-\frac{y}{\theta_2}} \quad , y>0 \ , \theta_2 > 0.$$

Derive the UMPU test for testing the hypothesis $H: \theta_1 = \theta_2$ against $A: \theta_1 > \theta_2$ at level of significance α.

20% 6. Let $\Omega = (0, \infty)$, and let the loss function be $L(\theta, a) = (\theta - a)^2$. Let the distribution of X be Poisson with parameter $\theta > 0$,

$$f(x;\theta) = e^{-\theta} \frac{\theta^x}{x!} \ , x = 0, 1, 2, \cdots$$

(a) Take the prior distribution of θ $g(\theta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} e^{-\theta/\beta} \theta^{\alpha-1}$,

Find the Bayes estimator.

(b) Find the minimax.