Probability

1. Show that the function

$$F(x, y) = \begin{cases} 0 & \text{for} \quad x+y < 1 \\ 1 & \text{for} \quad x+y \geq 1 \end{cases}$$

is not a joint distribution function. 5%

2. Suppose that X and Y are independent random variables. They are both uniformly distributed on $(0, 1)$. Find the probability that the roots of $\lambda^2 + 2X\lambda + Y = 0$ are real. 10%

3. X is called a lognormal random variable, if the $\log X = Y$ has a normal distribution $N(\mu, \sigma^2)$.
   (1) Find
      (a) the density of X. 5%
      (b) $E(X)$ and $Var(X)$. 10%
   (2) Let the X_i be independent lognormal random variables. Show that their product $X_1X_2\cdots X_n$ is also lognormal. 5%

4. Let X and Y be independent random variables with Cauchy density

$$f(x) = \frac{1}{\pi(1+x^2)}, \quad -\infty < x < \infty.$$

Show that $\frac{X+Y}{2}$ also has Cauchy density $f(x)$. 15%