I.(a) Use the Lagrange interpolation formula to find a polynomial f with real coefficients such that degree of $f \leq 3$ and f contains the points $(-1,-6)$, $(0,2)$, $(1,-2)$ and $(2,6)$. (5%)

(b) Let T be a linear transformation of the vector space over a scalar field F, id_V the identity transformation (i.e. $id_V(x) = x$ $\forall x \in V$) and $T^2 = 1$) the zero transformation. Show that $id_V - T$ is invertible. (10%)

II. Let $A = \begin{pmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & 2 \end{pmatrix}$. Find a suitable 3×3 matrix M such that $M^{-1}AM$ is in Jordan canonical form. (10%)

III What is the orthogonal complement of the subspace $W = \{(x,y,z) \mid x+y+z=0, x-y+z=0\}$ of $\mathbb{R}^3$. (10%)

IV. Find a unitary matrix P that diagonalizes $A = \begin{pmatrix} 2 & 1+i \\ 1-i & 3 \end{pmatrix}$ where $i^2 = -1$. (15%)