(離散數學)

(8%) 1. Let Q_+^* be the set of positive rational numbers, and $a \oplus b = ab/2$ for $a, b \in Q_+^*$. Prove that $< Q_+^*, \oplus >$ is a group.

(8%) 2. Define $n R m$ if and only if $nm >= 0$ for any integers n and m. Is R an equivalence relation? Why?

(8%) 3. Find the solution for the recurrence system:

$$\begin{cases} a_0 = 0, \ a_1 = 1 \\ a_n = a_{n-1} + a_{n-2} \quad \text{for } n > 1 \end{cases}$$

(8%) 4. Let $\Sigma = \{0,1\}$. Design a (deterministic) finite state machine that accepts the set of all strings over Σ which ends with 100.

(8%) 5. Let $\bar{G}$ be the complement of the graph G. Prove that if G is disconnected then $\bar{G}$ is connected.

(10%) 6. (a) Construct a recurrence procedure MAX to implement a divide and conquer strategy for finding the largest element in the entries $A(i)$, ---, $A(j)$ of an array A.

(b) Let $f(n)$ be the number of comparisons made between entries of n elements (n is a power of 2). Find the recurrence system which characterizes the complexity for MAX.