1. (a) Let Z be a random variable with finite first moment. Show that $f(b) = E|Z-b|$ is minimize when b = any median of the distribution of Z. 10%

   (b) Consider the loss function $L(\theta, a) = |\theta - a|$, X is a random variable having a uniform distribution on the interval $(0, \theta)$. Find a Bayes estimate of θ with respect to $g(\theta) = \theta e^{-\theta}$, $\theta > 0$. 10%

2. Let $X_1, \ldots, X_n$ be a sample from the uniform $U(0, \theta)$ population, where $0 < \theta < \infty$.

   (a) Show that $M_n = \max(X_1, \ldots, X_n)$ is a sufficient complete estimate of θ. 10%

   (b) Find the uniform minimum variance estimate of θ. 10%

3. Consider the simple linear regression model

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i, \quad i = 1, \ldots, n$$

   where β_0, β_1 are parameter, X_i are known constants, ϵ_i are independent $N(0, \sigma^2)$, σ^2 is a unknown parameter.

   (a) Find the maximum likelihood estimates of β_0, β_1 and σ^2. 10%

   (b) Derive a $100(1-\alpha)\%$ confidence interval of β_1. 10%

4. Let $X_1, \ldots, X_n$ be a sample from the normal $N(\mu, \sigma^2)$, where μ, σ^2 are unknown parameters.

   (a) Find the likelihood ratio (L.R.) test for $H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$ where μ_0 is a known constant. 10%

   (b) Find the L.R. test for $H_0: \sigma = \sigma_0$ vs $H_1: \sigma \neq \sigma_0$ where σ_0 is a known constant. 10%

5. (a) Define the Fisher's information (about θ). 10%

   (b) Let $I_{\tilde{X}}(\theta)$ be the Fisher's information (about θ) contained in the sample $\tilde{X} = (X_1, \ldots, X_n)$, and $I_T(\theta)$ be the Fisher's information (about θ) provided by T, where $T = T(\tilde{X})$ is a sufficient statistic of θ. Show that $I_T(\theta) = I_{\tilde{X}}(\theta)$. 10%