基礎數學 C

國立成功大學 81 學年度應數所考試(線性代數試題) 共 3 頁 第 1 頁

Part I

1. Let V and W be vector spaces over the field F and $\dim V$, the dimension of V, finite. Let $T: V \to W$ be a linear transformation of V into W. Prove that

$$\dim V = \dim \ker T + \dim T(V).$$

[Here $\ker T$ denotes the kernel of T and $T(V)$ denotes the image of T.] (15%)

2. Let V denote the vector space of functions that are linear combinations of e^x, xe^x, x^2e^x and e^{2x}. Define $T: V \to V$ by $T(f) = \frac{d}{dx}(f)$ for every $f \in V$. Find both a Jordan canonical form and a Jordan canonical basis for T. (20%)

3. Let $V = \{(x, y, z) \mid x, y, z \in \mathbb{C}\}$ be a vector space over the complex field $\mathbb{C}$ with the Euclidean inner product $u \cdot v = \sum_{i=1}^{3} u_i \overline{v_i}$ where $u = (u_1, u_2, u_3)$, $v = (v_1, v_2, v_3) \in \mathbb{C}^3 = V$. Apply the Gram-Schmidt process to transform the basis (i, i, i), $(0, i, i)$, $(0, 0, i)$ into an orthonormal basis. [Here $i = \sqrt{-1}$.] (15%)

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