國立成功大學八十一學年度應數所(乙)入學考試(常微分方程 試題)

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第 1 頁

1. Consider the equation $dx/dt = f(x)$ and suppose that $x=a$ is a critical point. Show that the constant equilibrium solution $u(t)=a$ is stable if $f'(a) < 0$ and unstable if $f'(a) > 0$. (15%)

2. (a) Show that $u(x,y)$ is a particular solution of the equation

$$P(x,y)u_x + Q(x,y)u_y = 0$$

if and only if $u=c$ is the general solution of the equation $Qdx-Pdy=0$, where c is a constant. (8%)

(b) Suppuse u is a solution for the partial differential equation given in (a). Show that $v=f(u)$ is also a solution of the equation, where f is a differentiable function. This solution is called the general solution. (2%)

(c) Find the general solution of the following equation

$$((x^2+y)^2+y)u_y + xu_x = 0. \quad (15\%)$$

3. Solve the following boundary problem by determining the appropriate Green's function and expressing the solution as a definite integral.

$$y'' + y = -f, \quad y'(0)=0, \ y(1)=0. \quad (20\%)$$

4. Consider the differential equation $3xy'' + y' - y = 0$.

(a) Show that $x=0$ is a regular singular point. (5%)

(b) Suppose that the solution is given by the form $\sum_{n=0}^{\infty} c_n x^{n+r}$. Determine the values of r and find the relations between c_n's. (20%)

5. For a vector $b \in R^n$, denote $|b| = (\sum_{j=1}^{n} b_j^2)^{\frac{1}{2}}$, where $b=(b_1,\dots,b_n)$. For a $n \times n$ matrix $A=(a_{ij})$, we denote $\|A\|_2 = (\sum_{ij} |a_{ij}|^2)^{\frac{1}{2}}$. Now consider the following equation $x'(t) = Ax(t)$. Let $x(t,a)$ denote the solution such that $x(0,a)=a$. Show that $|x(t,a)-x(t,b)| \leq \exp(\|A\|_2 t)\,|b-a|$. (15%)

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