1. (a) Let $X \sim b(n,\theta)$ (binormial distribution), $0 < \theta < 1$. Using the lòss function

$$L(\theta,d) = \frac{(\theta - d)^2}{\theta(1-\theta)}$$

Show that $\frac{X}{n}$ is a minimax estimate of θ. (10%)

(b) Let X, Y be independent with distributions $b(n,p_1)$, $b(n,p_2)$ respectively. An estimate is required of the difference $p_1 - p_2$, using loss function $L((p_1,p_2),d) = (p_1-p_2-d)^2$ where $|d| \le 1$. Find the Bayes estimate with respect to the prior distribution which assigns independent uniform distribution on $(0,1)$ to p_1 and p_2. (10%)

2. Let $X_1,\dots,X_n \sim b(1,p)$, $0 < p < 1$.

(a) Find the likelihood ratio test for $H_0 : p \le p_0$ vs $H_1 : p > p_0$ with α as the level of significance. (10%)

(b) Find the uniform minimum variance estimate of $p(1-p)$. (10%)

3. A computer center recorded the number of programs it maintained during each of 10 consecutive years :

Year	1	2	3	4	5	6	7	8	9	10
Number of program	430	480	565	790	885	960	1200	1380	1530	1591

Find a 95% prediction interval

(a) for the number of programs in the year 11. (10%)

(b) for the expected number of programs in the year 11. (10%)

(Hint : $t_{0.025}(8) = 2.306$ $t_{0.025}(9) = 2.262$ $t_{0.025}(10) = 2.228$
$t_{0.05}(8) = 1.86$ $t_{0.05}(9) = 1.833$ $t_{0.05}(10) = 1.812$)

4. Let X be a random variable.

(a) Find the most powerful test for $H_0 : X \sim N(0,1)$ vs $H_1 : X \sim C(0,1)$ (cauchy distribution) at level α. (10%)

(b) Find the power of the test. (10%)

5. Let X be amount of butterfat (in pounds) produced by 40 cows. Test the hypothesis that the distribution of X is normal with $\alpha = 0.01$ as the level of significance

16.93	18.79	14.62	13.98	15.79	12.39	13.20	16.08	13.97	16.16
16.12	17.81	18.74	15.99	13.32	13.63	16.40	13.76	16.58	15.25
18.97	18.36	15.04	18.79	18.08	17.32	16.32	17.54	18.05	14.20
18.04	13.00	13.25	12.43	16.56	14.12	20.55	16.75	13.29	18.23

(Hint : (i) By the table of the standard normal distribution, each set has probability 12.5% if we partition the real line $(-\infty,\infty)$ into 8 sets with the points $-1.15, -0.67, -0.32, 0, 0.32, 0.67, 1.15$.

(ii) $\bar{x} = 15.96$, $s_x = 2.144$.) (20%)

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