

Differential Equations.

(15 points each for the first two problems, 14 points each for the others)

1. For the undamped pendulum equation

$$L\theta'' + g \sin \theta = 0,$$

draw the phase plane and locate the attracting points. Note that a phase plane is a plot of the derivative of the dependent variable against the dependent variable in a second-order equation.

2. For the linear system of equations

$$\frac{dx}{dt} = 2x - y - 2, \quad \frac{dy}{dt} = 3x - 2y - 2,$$

find all critical points and investigate stability (asymptotically stable, stable, or unstable) of each.

3. Find the eigenvalues and eigenfunctions of the eigenvalue problem

$$X''(x) + \lambda X(x) = 0, \quad X'(0) = X(\pi) = 0.$$

4. Calculate by hand the approximating values of  $x(0.2)$  and  $y(0.2)$  of the initial value problem

$$\begin{aligned} x' &= 9x + 5y, & x(0) &= 1, \\ y' &= 3x + 2y, & y(0) &= 1, \end{aligned}$$

by using the Euler method with two steps of size  $h = 0.1$ .

5. Find the Laplace transform of the solution of the problem

$$x'' + 4x = \sin 2t, \quad x(0) = 1, \quad x'(0) = 4.$$

6. Find a second solution of

$$x^2 y'' - 3xy' + 3y = 0$$

in the form  $y_2 = u(x)y_1$ , given the first solution  $y_1 = x$ .

7. For the differential equation

$$x^2 y'' + xy' + (x^2 - \frac{1}{16})y = 0,$$

find the indicial equation and its roots at each regular singular point.