1. Find the following limits.
   (i)(5%) $\lim_{x\to 0} \frac{x^4}{\cos x - (1 - x^2)}$.
   (ii)(5%) $\lim_{n\to\infty} \int_0^{\frac{\pi}{2}} x^5 \sin(nx)dx$.
   (iii)(10%) $\lim_{n\to\infty} \sum_{i=1}^{n} \sum_{j=1}^{[\sqrt{n^2-i^2}]} \frac{i^2+j^2}{n^4}$, where $[\sqrt{n^2-i^2}]$ denotes the largest integer which is not larger than $\sqrt{n^2-i^2}$.

2. Find the following definite integrals.
   (i)(5%) $\int_0^2 \frac{x^2}{(x^3+1)^2} dx$
   (ii)(5%) $\int_1^e x^2 \ln x dx$
   (iii)(5%) $\int_0^2 \frac{(x+2)^2 + (x+1)^2}{(x+1)^2(x+2)} dx$
   (iv)(5%) $\int_{-\infty}^{\infty} H(x)H(\pi/2 - x)\sin x dx$, where $H(x) = \begin{cases} 1, & \text{if } x \geq 0; \\ 0, & \text{if } x < 0. \end{cases}$

3. Find the derivatives of the function $f(x)$ in (i) and (ii).
   (i)(5%) $f(x) = 5^x(\sin x)^2$.
   (ii)(5%) $f(x) = \int_{x^2}^{0} \sin(t^2)dt$.

4. (10%) Show that a function $f(x)$ with zero derivative on an open interval (a, b) must be a constant.

5. (15%) Using an approximation, $\frac{h}{3}(f(-h) + 4f(0) + f(h))$, for the intgral $\int_{-h}^{h} f(x)dx$ is called Simpson's rule. Show that Simpson's rule is exact as the function $f(x)$ is a third order polynomial.

6. (15%) Show that there exists a positive integer N for any real number M such that $\sum_{i=1}^{N} \frac{1}{i} > M$.

7. The equation $z = F(x, y) = 2x^2y^2$ represents a surface in the three-dimensional space where x, y, z are the Cartesian coordinates. A particle travels on the surface. The position of the particle is $(x, y, z) = (\cos t, \sin t, F(x, y))$ at any time t.
   (i)(5%) Find the directional derivative of $F(x, y)$ in the direction $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ on $x - y$ plane.
   (ii)(5%) Calculate the length of the path that the particle travels during the time interval $[0, 1]$.