1. Let $X$ be a discrete r.v. with density $f(x) = 2^{-x}$, $x = 1, 2, \ldots$
   (a) Find $EX$. (6%)
   (b) Let $Y = g \circ X$, where $g(n) = \dfrac{(-1)^{n+1}2^n}{n}$. Does $EY$ exist? Justify your answer. (6%)

2. Consider r.v. $X \sim N(0,1)$ and let $Y = e^X$.
   (a) Find the $n$th moment of $Y$, $EY^n$. (6%)
   (b) Does the moment generating function of $Y$ exist? Justify your answer. (6%)

3. Consider the discrete probability space with sample space $S = \{0, 1\}$ and equal probability mass $\frac{1}{2}$ at each point in $S$. Define

$$X_n(s) = \begin{cases} 0 & \text{if } s = 0 \\ 1 & \text{if } s = 1, \quad n = 1, 2, \ldots, \end{cases}$$

$$X(s) = \begin{cases} 1 & \text{if } s = 0 \\ 0 & \text{if } s = 1. \end{cases}$$

   (a) Does $X_n$ converge in distribution to $X$? Justify your answer. (7%)
   (b) Does $X_n$ converge in probability to $X$? Justify your answer. (7%)
   (c) Does $X_n$ converge in the $p$th mean to $X$? Justify your answer. (7%)

4. Let $X$ be any r.v., and suppose that the moment generating function of $X$, $M(t) = Ee^{tX}$, exists for every $t > 0$. Show that for any $t > 0$,

$$P\{tX > s^2 + \log M(t)\} < e^{-s^2}. \quad (10\%)$$

5. Let $X_n \xrightarrow{P} X$ and $g$ be a continuous function defined on $\mathbb{R}$. Show that $g(X_n) \xrightarrow{P} g(X)$ as $n \to +\infty$. (20%)

6. Let $X_1, \ldots, X_n$ be independent and identically distributed as Uniform $(0, a)$, $a > 0$, and let $X_{(1)}, \ldots, X_{(n)}$ denote the order statistics. Define $R = X_{(n)} - X_{(1)}$ and $V = \dfrac{(X_{(1)} + X_{(n)})}{2}$.
   (a) Find the joint p.d.f. of $(R, V)$. (10%)
   (b) What is the distribution of $\dfrac{R}{a}$? (8%)
   (c) Find the marginal p.d.f. of $V$. (7%)