1. Let $V = \{(x, y, z) \mid x - 2y + 3z = 0\}$.

   (a) Show that V is a subspace of $\mathbb{R}^3$. (5%)

   (b) Find an orthonormal basis for V. (5%)

   (c) Find the orthogonal projection of the vector $v = (1, 2, 3)$ on V. (5%)

   (d) Let P be the orthogonal projection from $\mathbb{R}^3$ onto V. Find $P(x, y, z)$. (5%)

2. Let $A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 0 & -1 \\ 0 & 1 & 3 \end{pmatrix}$, find A^{20}. (15%)

3. Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation and $N(T)$ the null space of T.

   (a) Show that $N(T) \subseteq N(T^2)$. (5%)

   (b) Give an example of T so that $N(T) \neq N(T^2)$. (5%)

   (c) Prove that if $N(T) = \{0\}$, then $N(T^2) = \{0\}$. (8%)

   (d) Prove that there exists an integer k such that $N(T^k) = N(T^{k+1})$. (7%)

4. Let $A = \begin{bmatrix} 0 & a \\ b & c \end{bmatrix}$, $abc \neq 0$. Find elementary matrices E_1, E_2, E_3 and E_4 such that $A = E_4 E_3 E_2 E_1$. (10%)

5. Suppose A is a 3×3 matrix with $\operatorname{trace}(A) = \det A = 0$. Show that $A^3 = \alpha A$ for some constant α. (10%)

6. Let $V = P_2(\mathbb{R})$ with the inner product $\langle f, g \rangle = \int_0^1 f(t) g(t)\, dt$. If $T(f) = 2f' + f$ for $f \in P_2(\mathbb{R})$. Find $T^*(f_0)$ where $f_0(x) = 5 - 2x + 4x^2$ and T^* is the dual operator of T. (10%)

7. Suppose that A and B are diagonalizable matrices. Prove or disprove that A is similar to B if and only if A and B are unitarily equivalent. (10%)