1. Let $X_1, \cdots, X_n$ be independent and identically distributed random variables (i.i.d.r.v.) which is exponentially distributed with parameter $\alpha = 0.001$; that is its pdf is $f(x) = 0.001e^{-0.001x}$, $x > 0$.
(a) What distribution does $\overline{X}$ have? where $\overline{X} = \dfrac{X_1 + \cdots + X_n}{n}$;
(b) Find $\lim_{n\to\infty} \overline{X}$. (20%)

2. Let $\{N_t\}_{t>0}$ be a Poisson process with rate $\lambda > 0$ (N_t has poisson distribution $P(\lambda t)$). $T_k = \inf\{t : N_t = k\}$ is the time of the kth arrival where $k \in \mathbb{N}$. Find the p.d.f. of T_k. (12%)

3. Let $X_1, X_2 \cdots$ be a sequence of i.i.d.r.v.'s with $p\{X_1 = 1\} = p$, $p\{X_1 = 0\} = 1 - p$ where $0 < p < 1$. Let $S_n = \sum_{k=1}^{n} X_k$ and $T_k = \min\{n : S_n = k\}$ where $k \in \mathbb{N}$. Find the distribution of T_k. (12%)

4. Suppose that $Y_\lambda \stackrel{d}{=} P(\lambda)$, $\lambda > 0$. Prove that $\dfrac{Y_\lambda - \lambda}{\sqrt{\lambda}} \xrightarrow{d} N(0,1)$ as $\lambda \to \infty$, where $\xrightarrow{d}$ denotes convergence in distribution.
(Hint: use characteristic functions) (12%)

5. Calculate $E[\dfrac{1}{X+1}]$ when
(a) $X \stackrel{d}{=} P(\lambda)$
(b) $X \stackrel{d}{=} B(n,p)$
binormial distribution with parameter n and p. (20%)

6. If $E(X) < \infty$ and if $P\{X \geq m\} \geq \frac{1}{2}$, $P\{X \leq m\} \geq \frac{1}{2}$ for some $m \in \mathbb{R}$. Prove that $E|X - m| \leq E|X - a|$ for all $a \in \mathbb{R}$. (12%)

7. Let $X_1, X_2, \cdots$ be i.i.d.r.v's and $S_n = \sum_{i=1}^{n} X_i$. Prove that $E[\dfrac{S_m}{S_n}] = \dfrac{m}{n}$ if $m \leq n$. (12%)