1. Let X be a r.v. distributed as $P(\lambda)$, $\lambda > 0$ and set $E = \{2(k-1) \mid k \in \mathbb{N}\}$, $O = \{2k-1 \mid k \in \mathbb{N}\}$.
   (a) Find the values of $S_E = \sum_{x \in E} \frac{\lambda^x}{x!}$ and $S_O = \sum_{x \in O} \frac{\lambda^x}{x!}$. (6%)
   (b) Use (a), calculate the probabilities: $P(X \in E)$ and $P(X \in O)$. (6%)
   (c) Suppose that the value $x = 0$ cannot be observed. Find the p.d.f. (or p.m.f.) of the truncated r.v., its mean, and its variance. (8%)

2. (a) Prove that if $\mathrm{Var}(X) = 0$, then $P(X = EX) = 1$. (8%)
   (b) Let X_j, $j = 1, \ldots, n$ be i.i.d. r.v.'s with common variance σ^2. Also let a_j, $j = 1, 2, \ldots, n$ be real numbers such that $\sum_{j=1}^{n} a_j = 1$, and let $S = \sum_{j=1}^{n} a_j X_j$. Show that the variance of S is least if we choose $a_j = \frac{1}{n}$, $j = 1, \ldots, n$. (7%)

3. Let (X, Y) be jointly distributed with density function

$$f(x,y) = \begin{cases} x+y, & \text{if } 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find the m.g.f. of (X, Y). (8%)
   (b) Are X, Y independent? If not, find the correlation coefficient between X and Y. (7%)

4. Let $X_1, X_2, \ldots, X_n$ be i.i.d. $U(0,1)$ r.v.'s. Find the p.d.f. of $Y_n = (\prod_{i=1}^{n} X_i)^{\frac{1}{n}}$. (15%)

5. Let $X_1, X_2, \ldots, X_n$ be i.i.d. $N(0,1)$ r.v.'s. Define

$$U_n = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} X_i, \quad V_n = \frac{1}{n} \sum_{i=1}^{n} X_i^2, \quad \text{and}$$

$$W_n = \frac{U_n}{V_n}.$$

   (a) Find the distribrtions of U_n and V_n. (10%)
   (b) Find the limiting distribution of W_n. (10%)

6. Let X_j, $j = 1, \ldots, n$ be independent r.v.'s having the Cauchy distribution with parameters $\mu = 0$ and $\sigma = 1$, $C(0,1)$. Find the distribution of $\overline{X}_n = \frac{1}{n} \sum_{j=1}^{n} X_j$, and show that there is no finite constant c for which $\overline{X}_n \xrightarrow{P} c$. (15%)

   Hint. The p.d.f. of $C(\mu, \sigma)$ is given by

$$f(x; \mu, \sigma) = \frac{\sigma}{\pi} \cdot \frac{1}{\sigma^2 + (x-\mu)^2}, \ x \in \mathbb{R}, \ \mu \in \mathbb{R}, \ \sigma > 0.$$