

系所組別：光電科學與工程學系甲組

考試科目：近代物理

考試日期：0226，節次：1

1. (6 %) A light source with wavelength λ illuminates a metal and ejects photoelectron of a maximum kinetic energy of 1 eV. Another light source with wavelength $\lambda/2$ ejects photoelectron of a maximum kinetic energy of 4 eV. What is the work function (in eV) of the metal?
2. (4 %) (a) Show that the energy-momentum relationship given by $E^2 = p^2 c^2 + (mc^2)^2$ follows from the expression $E = \gamma mc^2$ and $p = \gamma m v$, where γ is
$$\gamma \text{ is } \frac{1}{\sqrt{1 - v^2/c^2}}$$
- (4 %) (b) The total energy of a proton is three times of its rest energy. Find the proton's rest energy (in eV) and its moving speed.
- (4 %) (c) Determine the kinetic energy of the proton in electron volts.
- (6 %) (d) What is the proton's momentum (in eV/c)?
[Mass of proton $m_p = 1.67 \times 10^{-27}$ kg, $1 \text{ eV} = 1.6 \times 10^{-19}$ J, $\sqrt{2} = 1.414$]
3. (6 %) Calculate the de Broglie wavelength (in meter) of a 74-kg person who is running at a speed of 5.0 m/s. [$h = 6.63 \times 10^{-34}$ J·s]
4. (4 %) (a) Use Stefan's Law to calculate the total power radiation per unit area by a filament at a temperature of 3000 K assuming the filament is an ideal radiator. [$\sigma = 5.7 \times 10^{-8}$ W/m²K⁴]
- (4 %) (b) Assume that the sun radiates as a black body with a surface temperature of 5800 K. Use Wien's displacement law to show the peak (in nm) of the solar spectrum. [Wien's displacement constant = 2.898×10^{-3} mK]
- (4 %) (c) What is the average energy \bar{E} (in KT) of an oscillator that has a frequency $h\nu = KT$ according to Planck's calculation? [$e = 2.718$]
5. (4 %) (a) What is the electronic configuration (ground state) of element Al (aluminum) whose atomic number $Z = 13$.
- (4 %) (b) What are the quantum numbers (n, l, m, s) for the electron at the out most orbital?

(下一頁還有試題)

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6. A free particle of mass m with wave number k_1 is traveling to the right. At $x = 0$, the potential jumps from zero to V_0 and remains at this value for positive x .
- (4 %) (a) If the total energy is $E = 2V_0$, what is the wave number k_2 in the region $x > 0$? Express your answer in terms of k_1 and V_0 .
- (8 %) (b) Calculate the reflection coefficient R and the transmission coefficient T .
- (4 %) (c) What is the probability current carried by this particle?
7. (8 %) What are the possible values of the total angular momentum of an electron in a d state (the $\ell = 2$ state)? What are the angles between the spin and the orbital angular momentum? Please also compute the magnitude of the spin and orbital angular momenta.
- (8 %) In a carbon atom, only the two $2p$ electrons contribute to its angular momentum. The ground state of this atom is 3P_0 , and the first four excited states, in order of increasing energy, are 3P_1 , 3P_2 , 1D_2 , 1S_0 . (a) Give the L , S , J values for each of these five states. (b) Why do you think the 3P_0 state is the ground state?
8. (3 %) (a) If $\psi_\alpha(l)$ represents particle 1 in the α state, use this representation to write normalized wave function for a system of three particles, when these three particles are Fermions.
- (3 %) (b) What is Fermi-Dirac distribution?
- (6 %) (c) A metal of volume V contains N electrons. The number of quantum states in an energy interval E to $E+dE$ is given by
- $$g(E)dE = \frac{8\sqrt{2}\pi V m^{3/2}}{h^3} \sqrt{E} dE. \text{ Please derive the Fermi energy.}$$
- (6 %) (d) Following (b) and (c), please derive the average energy of an electron at $T = 0$ K.