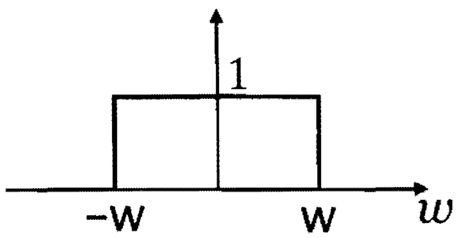
1. For the initial value problem $y'' + 4y = t$, $y(0) = 1$, $y'(0) = -2$
   (1) Find the Green's function $G(t,\tau)$ for the differential equation using Laplace transform. (10%)
   (2) Using the result of (1), solve the initial value problem using $G(t,\tau)$. (10%)

2. For the Strum-Liouville problem $(1-x^2)y'' - 2xy' + n(n+1)y = 0$ on the interval [-1,1]
   (1) Find its eigenvalues and eigenfunctions. (5%)
   (2) The set of eigenfunctions are orthogonal with respect to a weight function $p(x)$ on the interval [-1,1], what is $p(x)$? (5%)

3. The Fourier transform $\hat{f}(\omega)$ of a function $f(x)$ is given in the figure below, find $f(x)$. (10%)



4. Consider a discrete function $f[k] = \cos(w_o k)$ ($k = 0, \pm1, \pm2, \ldots$), $w_o = 2\pi/N$
   (1) What is the fundamental period of $f[k]$? (5%)
   (2) The discrete Fourier transform of $f[k]$ can be written as $\sum_{n=0}^{N-1} c_n e^{in\omega_0 k}$, find c_1 and c_2. (5%)

5. A function w(x,y) is continuous and has continuous first and second partial derivatives in a domain of the xy-plane containing a region R. Let R be a closed bounded region in the xy-plane whose boundary C consists of finitely smooth curve. The vector $\vec{n}$ is a unit normal vector to C and has $\vec{r}' \cdot \vec{n} = 0$, where $\vec{r}$ is the parametric representation of C, ds is the linear element of C and $\vec{r}' = d\vec{r}/ds$. Using Green's Theorem, show that
$\iint_R (\nabla^2 w) dxdy = \oint_C \frac{\partial w}{\partial n} ds$. (15%)

6. Verify that $u(x,y)=\ln|z|$ is harmonic, and find a corresponding analytic function $f(x,y)=u(x,y)+iv(x,y)$ with complex number $z=x+iy$. (12%)

7. Find the center and the radius of convergence of the power series $\sum_{n=0}^{\infty}\frac{n!}{n^n}(z+1)^n$. (8%)

8. Find the Cauchy principal value of $\int_{-\infty}^{\infty}\frac{\cos(mx)}{x^4-1}dx$, where m is a constant. (15%)