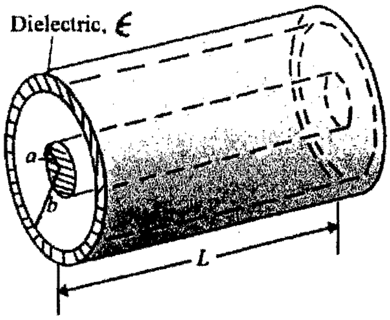
1. A cylindrical capacitor consists of an inner conductor of radius a and an outer conductor whose inner radius is b, as shown below. The space between the conductors is filled with a dielectric of permittivity ε, and the length of the capacitor is L. Assume the outer conductor is grounded and that the inner conductor is maintained at V_0. (15%)

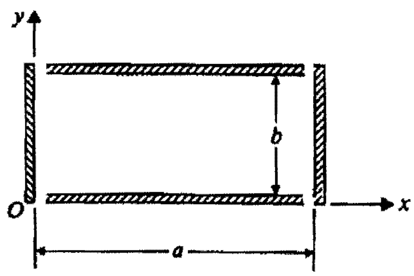
   (a) Determine the electric field intensity $\bar{E}(a)$ at the surface of the inner conductor. (7%)

   (b) With the inner radius, b, fixed, find a so that $\bar{E}(a)$ is minimized and the value of $\bar{E}(a)$. (5%)

   (c) Determine the capacitance under the conditions of part (b). (3%)



2. Consider the rectangular region shown below as the cross section of an enclosure formed by four conducting planes. All planes are assumed to be infinite in extent in the z-direction. Determine the electric potential distribution within this region if the left and right planes are grounded and the top and bottom plates are kept at constant potentials V_1 and V_2, respectively. (17%).

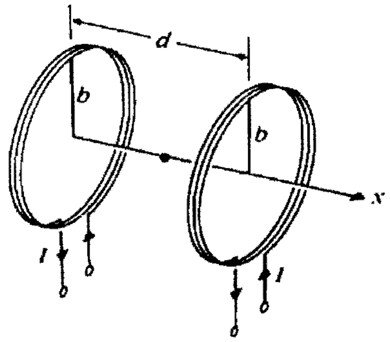


3. Helmholtz coils shown below are used to obtain an approximately uniform magnetic field in the midpoint region. They consist of two identical coaxial coils separated by a distance d. Each coil has a radius of b and contains N turns. A current I flows in each coil in the same direction. (18%)

(a) Find the magnetic flux density at a point on the axis between the coils. Start from finding the magnetic flux density at a point on the axis of one coil turn. (8%)

(b) Find the condition(s) required to obtain an approximately uniform magnetic field in the midpoint region. Start from Taylor-expanding the expression of the magnetic flux density found in (a) around the midpoint to the second order. (6%)

(c) What is the magnetic flux density at the midpoint if the condition(s) found in (b) is/are satisfied? (4%)



4. (a) Write down the ***time varying*** *Maxwell equations* in differential form with current J and charge density ρ. (5%)

(b) From (a), derive the non-homogeneous *wave equation* for vector potential A and scalar potential V. Specify the *gauge* you used. (5%)

5. The electric field intensity of a linearly polarized uniform plane wave propagating in the +z direction in seawater is $\mathbf{E} = \hat{x}100\cos\left(2\times10^{7}\pi t\right)\left[\frac{V}{m}\right]$ at $z = 0$. The seawater can be considered as *a good conductor* with $\varepsilon_r = 72$, $\mu_r = 1$ and $\sigma = 4$ (S/m). $\mu_0 = 4\pi\times10^{-7}$, $\varepsilon_0 = (1/36\pi)\times10^{-9}$. Determine the attenuation constant α, phase constant β, intrinsic impedance η_c, phase velocity u_p, wavelength λ, and skin depth δ. (You are required to write the expression and numbers) (15%)

6. TM modes propagate in the z-direction of a dielectric slab waveguide with dielectric material ε_d of thickness d surrounding by air ε_0. From $\frac{d^2}{dy^2}E_z^0 + h^2E_z^0 = 0$ where $h^2 = \gamma^2 + k^2 = (j\beta)^2 + \omega^2\mu\varepsilon$, E_z has a form $E_z^0(y) = E_0\sin(k_y y) + E_e\cos(k_y y)$ $|y| \le \frac{d}{2}$ inside the dielectric slab and

$$E_z^0(y) = \begin{cases} C_u e^{-\alpha\left(y-\frac{d}{2}\right)} & y \geq \frac{d}{2} \\ C_l e^{+\alpha\left(y+\frac{d}{2}\right)} & y \leq \frac{d}{2} \end{cases}$$ outside the dielectric slab waveguide. (15%)

(a) Find $k_y^2 + \alpha^2 = ?$ (3%)

(b) Using the boundary condition at y = d/2, -d/2 for E_z, find the expression of E_z^0 E_y^0 H_x^0 both inside (-d/2 < y < d/2) and outside (y > d/2) for odd TM mode and even TM mode. (5%)

(c) With H_x boundary condition at y = d/2, -d/2, we can get $\frac{\alpha}{k_y} = \frac{\varepsilon_0}{\varepsilon_d}\tan(\frac{k_y d}{2})$ for odd TM mode and $\frac{\alpha}{k_y} = -\frac{\varepsilon_0}{\varepsilon_d}\cot(\frac{k_y d}{2})$ for even TM mode. What is the cutoff frequency for odd TM1 and even TM_1 mode for d = 1um and $\varepsilon_d = 4$ $(1/\sqrt{\mu_0\varepsilon_0} = c = 3\times10^8 m/s)$. (3%)

(d) If we put a PEC plate at y = 0 , what are the cut-off frequency of the first f_{c1} and second f_{c2} modes. (4%)

7. If the cubic cavity resonator with size a is cut by half as a right triangular cavity resonator. What is the resonate frequency of the first mode? (Hint: this can be considered as a superposition of two rectangular cavity modes, such that all the parallel E fields at the edges of the triangle vanish.) (10%)