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A lot of useful physical constants are shown as below:

Speed of light in vacuum	$c = 3.00 \times 10^{8}$ m s^{-1}
Planck constant	$h = 6.63 \times 10^{-34}$ J s
Planck constant divided by 2π	$\hbar = 1.055 \times 10^{-34}$ J s
Electronic charge (absolute value)	$q_e = 1.602 \times 10^{-19}$ C
Fine structure constant	$\alpha = q_e^2/(4\pi\varepsilon_0 \hbar c) = e^2/(\hbar c) = 1/137$
Electron mass	$m_e = 9.11 \times 10^{-31}$ kg $= 0.511$ MeV c^{-2}
Proton mass	$m_p = 1.67 \times 10^{-27}$ kg $= 938$ MeV c^{-2}
Bohr magneton	$\mu_B = q_e\hbar/(2m_e) = 5.79 \times 10^{-5}$ eV T^{-1}
Nuclear magneton	$\mu_N = q_e\hbar/(2m_p) = 3.15 \times 10^{-8}$ eV T^{-1}
Bohr radius	$a_0 = \hbar^2/(m_e e^2) = 0.529 \times 10^{-8}$ m
Rydberg constant	$R_\infty = m_e e^4/(2\hbar^2) = 13.61$ eV
Boltzmann constant	$k_B = 1.38 \times 10^{-23}$ J K^{-1}
Electron volt and temperature	1 eV $= 1.602 \times 10^{-19}$ J $= k_B \times 11\,600$ K
Gravitational constant	$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

1. (5%) Describe "Compton scattering". Tell whether it illustrates the wave or particle nature of light and why.

2. (5%) A laser can produce a light pulse that is 5cm long. What is the minimum dispersion (error in wavelength) of such a pulse if the laser operates at 330nm (a nitrogen laser)?

3. (5%) When light of wavelength λ_1 is incident on the cathode of a photoelectric tube, the maximum kinetic energy of the emitted electrons is 1.8eV. If the wavelength is reduced to $\lambda_1/2$, the maximum kinetic energy of the emitted electrons is 5.5eV. Find the work function ϕ of the cathode material.

4. (20%) The wave function for a particle in a certain potential is given by $\psi(x) = \begin{cases} 0; & if\ x < 0 \\ Axe^{-ax}; & if\ x \geq 0 \end{cases}$,

   where a is given.

   (a) Write the equation that you would use to evaluate the constant A. (b) Write the equation that you would use to evaluate the average position of this particle assuming A is known. (c) Write the equation that you would use to evaluate the average of the square of the momentum for this particle again assuming A is known. (d) Sketch approximately the potential that could give such a wave function. Show approximately the level of the total energy on that potential.

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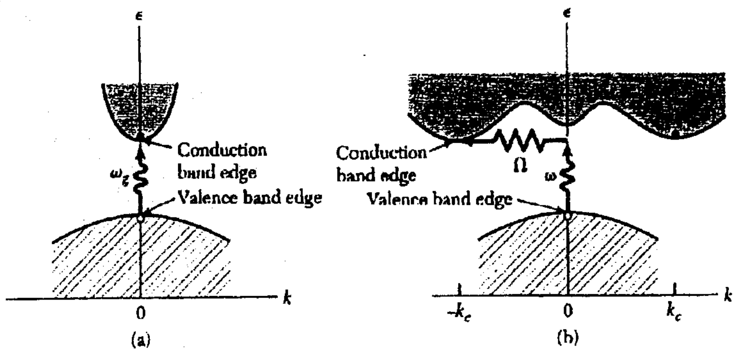
5. (15%) An electron is captured in a potential of the form: V=∞ for $x \le 0$ and $x \ge a$, V=0 for $0 < x < a/2$ and V=V_0 for $a/2 < x < a$. Draw the potential, the wave function for the ground state where E<V_0, and the second excited state where E>V_0.

6. (10%) When sitting in front of a color TV with a 25 kV picture tube potential, you have an excellent chance of being irradiated with X-rays. (a) What process produces most of the X-ray flux? (b) For the resulting continuous distribution, calculate the shortest wavelength X-ray.

7. (15%) (a) Please briefly illustrate "What is the free electron Fermi gas?" (b) Show that the kinetic energy of a three-dimensional gas of N free electrons at 0 K is $U_0 = \frac{3}{5} N\varepsilon_F$. The Fermi energy ε_F is defined as the energy of the topmost filled level in the ground state of the N electron system.

8. (10%) There are two different diagrams of energy band gap are shown as below.



Please compare what is difference between them, and illustrate what physical conditions are required when transitions occurring in detail.

9. (15%) According to quantum mechanics, electromagnetic radiation of frequency ν can be regarded as consisting of photons of energy hν. (a) What is the energy range of visible photons (400 nm to 700 nm)? (b) At a given power of an electromagnetic wave, do you expect a classical wave description to work better for radio frequencies, or X-rays? Why?