系所組別：光電科學與工程學系甲組
考試科目：近代物理 考試日期：0223，節次：1

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

(1) (10 %) (Energy and Momentum in Relativity)

(1-A) (5 %) please derive $E^2=(mc^2)^2+p^2c^2$. (Total Energy $E=mc^2/(1-v^2/c^2)^{1/2}$; $p=mv/(1-v^2/c^2)^{1/2}$)

(1-B) (5 %) Please reduce the relation of the E and P to $E=p^2/2m+mc^2$ for a non-relativistic case, for $V<<c$.

(2) (10 %) (photoelectric effect)

(2-A) (4 %) Please explain photoelectric effect. ($h\nu=KE_{max}+\phi$)

(2-B) (3 %) Please find the wavelength and frequency of a 200-MeV photon.

(2-C) (3 %) The maximum wavelength for photoelectric emission in tungsten is 230 nm. What wavelength of light must be used in order for electrons with a maximum energy of 1.5 eV to be ejected?

3. (10 %) (Uncertainty Principle)

(3-A) (3 %) An hydrogen atom is 5.3×10^{-11} m in radius. Use the uncertainty principle to estimate the minimum energy an electron can have in this motion.

(3-B) (3 %) Compare the uncertainties in the velocities of an electron and a proton confined in a 2.00-nm box.

(3-C) (4 %) Explain why the matter wave of a particle is represented by a wave packet rather than a single wave?

4. (10 %) (Atom and Atomic Spectrum)

(4-A) (5 %) In the Bohr model, the electron is in constant motion. How can such an electron have a negative amount of energy?

(4-B) (5 %) An electron collides with a hydrogen atom in its ground state and excites it to a state of n=2. How much energy was given to the hydrogen atom in the inelastic collision?

5. (10 %) (The Lasers)

(5-A) (6 %) Please use the energy level diagram to explain the "absorption", "stimulated emission", and "spontaneous emission".

(5-B) (4 %) Please describe the 3-level laser and 4-level laser systems.

(背面仍有題目,請繼續作答)

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6. Please explain "tunnel effect". (5%) The scanning tunneling microscope (STM) is based on this effect. For a one dimension quantum barrier, the approximate transmission probability is

$$T = e^{-2\frac{\sqrt{2m(U-E)}}{\hbar}L}$$

If the electron energy is 1 eV, the barrier is 10 eV high and 1 nm wide, please estimate the transmission probability. (5%)

7. For the hydrogen atom, if the wave function is φ, what is the actual probability of finding it in the infinitesimal volume element dV. (5%) If the electron is on 1s orbit ($\varphi = \frac{e^{-r/a_0}}{\sqrt{\pi a_0^{3/2}}}$), please show the expectation value of $1/r$. (5%)

8. What is the "spin-orbit coupling" in an atom? (5%) If the spin-orbit energy shift is

$$\langle V_{SL} \rangle = E_0 \frac{Z^4\alpha^2}{n^3} \frac{j(j+1)-l(l+1)-\frac{3}{4}}{l(l+1)(2l+1)}$$

, for $n=2$ and $l=1$ in the hydrogen atom, please estimate the energy difference of the two shifts. (5%)

9. What is "Fermi energy"? (5%) Please draw the distribution function under the temperature $T=0$, $T \ll T_F$, $T \gg T_F$ where $T_F \equiv \frac{\varepsilon_F}{k_B}$. (5%)

10. (a) Please prove that for the free electron system, the Fermi energy is $\varepsilon_F = \frac{\hbar^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$ where $\frac{N}{V}$ is the density of free electrons. (5%)

(b) Please use (a) to estimate the Fermi energy of gold (density is 19.3 g/cm^3, atomic weight: 196.96) (5%)

m: electron mass = 9.109x10^{-31} kg

α: fine structure constant =1/137

E_0: Rydberg energy unit = 13.6 eV

h: Planck's constant =6.626x10^{-34} J-s

eV=1.602x10^{-19} J