系所組別：光電科學與工程學系甲，乙組
考試科目：電磁學
※ 考生請注意：本試題不可使用計算機 。 請於答案卷（卡）作答，於本試題紙上作答者，不予計分。

1．Find the force between a charged circular loop of radius $b$ with uniform charge density $\rho_{l}$ and a point charge $Q$ located on the loop axis at a distance $h$ from the plane of the loop．What is the force when $h \gg b$ ， and when $h=0 . \quad(14 \%)$ ．

2．An uncharged conducting sphere of radius $b$ is placed in an initially uniform electric field $\bar{E}_{0}=\bar{z} E_{0}$ ．If $V(R=b, \theta)=V_{0}$ ，determine the potential distribution $V(R, \theta)$ and the electric field intensity $\bar{E}(R, \theta)$ after the introduction of the sphere． （20\％）

3．An infinitely long solenoid with air core having a radius $b$ and $n$ closely wound turns per unit length is shown below．The windings are slanted at an angle $\alpha$ and carry a current $l$ ．Determine the magnetic flux density both inside and outside the solenoid． （16\％）

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4．（ $15 \%$ ）Consider in metal with free electron charge $q_{e}$ ，mass $m_{e}$ and density $N_{e}$ ．Assume the effective collision frequency of electrons in metal is $v$ ．Please derive the relative permittivity of the metal（the Drude model）．

5．Consider a plane wave incident on a planar boundary at $\mathrm{x}=0$ from a dielectric medium with $\mu_{0}$ and $\varepsilon=9 \varepsilon_{0}$（region I）upon another dielectric medium with $\mu_{0}$ and $\varepsilon_{t}$（region II）．The right－hand circularly polarized incident electric field is

$$
\vec{E}_{i}=E_{0}(\sqrt{3} \hat{x}+\hat{z}) \cos \left(k_{x} x-k_{z} z-\omega t\right)+2 \hat{y} \sin \left(k_{x} x-k_{z} z-\omega t\right)
$$

where $E_{0}$ is a real constant．The reflected field is

$$
\vec{E}_{r}=E_{0}\left[2 R^{T E} \hat{y} \sin \left(k_{x} x+k_{z} z-\omega t\right)+R^{T M}(-\sqrt{3} \hat{x}+\hat{z}) \cos \left(k_{x} x+k_{z} z-\omega t\right)\right.
$$

（ $5 \%$ ）（a）What is the incident angle？
（ $5 \%$ ）（b）For $k_{x}=2 \pi / m$ ，find the frequency $(\mathrm{Hz})$ and wavelength（ $m$ ）in region I．
$(5 \%)(c)$ Find the value of $\varepsilon_{t}\left(0<\varepsilon_{t} / \varepsilon_{0}<\infty\right)$ for which the reflected wave is linearly polarized．

6．In an air－filled rectangular waveguide with dimensions $a=3 \sqrt{2} \mathrm{~cm}$ and $b=a / 2$ ，the guided wave is given by

$$
\begin{aligned}
& \bar{E}=\hat{y} E_{0} \sin \left(\frac{\pi}{a} x\right) \sin \left(\frac{\pi}{a} z-\omega t\right) \\
& \bar{H}=\hat{x} H_{0} \sin \left(\frac{\pi}{a} x\right) \sin \left(\frac{\pi}{a} z-\omega t\right)+\hat{z} H_{0} \cos \left(\frac{\pi}{a} x\right) \cos \left(\frac{\pi}{a} z-\omega t\right)
\end{aligned}
$$

where $E_{0}$ and $H_{0}$ are real constants．
（a）（ $4 \%$ ）What is the mode for this wave？Indicate the mode and the mode numbers $m$ and $n$ ．
（b）$(4 \%)$ What is the frequency？
（c）$(4 \%)$ What is the phase velocity in $\hat{z}$ direction in terms of the light speed c ？
（d）$(4 \%)$ What is the cutoff frequency of this mode？
（e）$(4 \%)$ If the waveguide is used as a rectangular cavity resonator for frequency $f=5 \mathrm{GHz}$ by closing the ends at $\mathrm{z}=0$ and $\mathrm{z}=d$ using perfectly conducting plates，what is the value of $d$ for the lowest mode？ Indicate this lowest mode and the mode numbers $\mathrm{m}, \mathrm{n}$ and p ．

