

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) Solve $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 3y = 0$

2. (10%) Given

$$\mathbf{A} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$

$$\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

find the expression of a unit vector \mathbf{C} that is perpendicular to both of \mathbf{A} and \mathbf{B} .

3. (10%) Find the general solution of the following system of differential equations.

$$\frac{dx}{dt} = x + 2y + z$$

$$\frac{dy}{dt} = 6x - y$$

$$\frac{dz}{dt} = -x - 2y - z$$

4. (10%) Use the Laplace transform to solve the following system of differential equations.

$$\frac{d^2 x}{dt^2} + \frac{dx}{dt} + \frac{dy}{dt} = 0$$

$$\frac{d^2 y}{dt^2} - \frac{dy}{dt} - 2 \frac{dx}{dt} = 0$$

$$x(0) = 1, x'(0) = 0$$

$$y(0) = -1, y'(0) = 1$$

5. (10%) Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = y^3 \hat{\mathbf{i}} - x^3 \hat{\mathbf{j}} + 9z^3 \hat{\mathbf{k}}$, C is the trace of the cylinder $x^2 + y^2 = 1$ in the plane $y + z = 2$. Assume C is orientated counterclockwise as viewed from above.

6. (10%) Find the complex Fourier series of f on the given interval:

$$f(x) = \begin{cases} 0, & -\frac{1}{2} < x < 0 \\ 1, & 0 < x < \frac{1}{4} \\ 0, & \frac{1}{4} < x < \frac{1}{2} \end{cases}$$

7. A thin wire coinciding with the x-axis on the interval $[-L, L]$ and with thermal diffusivity k is bent into the shape of a circle so that the ends are joined.

(a) (5%) List the partial differential equation describing the temperature distribution.

(b) (15%) Find the temperature $u(x, t)$ under the following conditions:

$$u(-L, t) = u(L, t), t > 0$$

$$\frac{\partial u}{\partial x} \Big|_{x=-L} = \frac{\partial u}{\partial x} \Big|_{x=L}, t > 0$$

$$u(x, 0) = f(x), -L < x < L$$

8. Expand $f(z) = \frac{1}{z(z-4)}$ in a Laurent series valid for

(a) (5%) $1 < |z|$

(b) (5%) $0 < |z-1| < 1$

9. (10%) Evaluate $\oint_C \frac{\tan z}{z} dz$, where the contour C is the circle $|z-1| = 2$.