※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%)

Find the Fourier transforms of

(a) $f(x) = \begin{cases} 1, |x| < a \\ 0, |x| > a \end{cases}$,

(b) $p(x) = \frac{1}{x^2+a^2}$ by making use of the standard integral $\int_{-\infty}^{\infty} \frac{\cos \omega x}{x^2+a^2} dx = \frac{\pi}{a} e^{-|\omega|a}, (a > 0)$.

2. (5%)

There is one matrix **A** which has eigenvalues 1 and -1 with respect to its corresponding eigenvectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Please show what is the matrix **A**.

3. (5%)

A triangle is composed of three vectors of $\vec{a}$, $\vec{b}$, and $\vec{c}$. Please prove the law of cosines for a triangle by means of vector analysis.

4. (15%)

Please make fully descriptions what is Green's Theorem and write out the equation along with a drawing illustration.

5. (5%)

A rotation $\varphi_1 + \varphi_2$ about the z-axis is carried out as two successive rotations φ_1 and φ_2, each about the z-axis. Use the matrix representations of the rotations to derive the trigonometric identities

$$\cos(\varphi_1 + \varphi_2) = \cos\varphi_1 \cos\varphi_2 - \sin\varphi_1 \sin\varphi_2,$$
$$\sin(\varphi_1 + \varphi_2) = \sin\varphi_1 \cos\varphi_2 + \cos\varphi_1 \sin\varphi_2.$$

6. (10%)

(a) Find the Fourier series representation of

$$f(x) = \begin{cases} 0, -\pi < x \leq 0 \\ x, 0 \leq x < \pi \end{cases}.$$

(b) From the Fourier expansion show that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots.$$

7. (15%)Find the integral $\int_0^{2\pi} \frac{d\theta}{(a+b\cos\theta)^2} = ?$ $(where\ 0<b<a)$ by Residue theorem

8. (15%) Find the solution of y(x) for $\frac{d^2y}{dx^2}+\frac{2}{x}\frac{dy}{dx}-\frac{l(l+1)}{x^2}y=\delta(x-a)$ $[0\le x<\infty]$

where $a>0$, l: positive integer, and y(0)=y(∞)=0

9. (10%) Use Laplace transform to solve $y'''-y'=\sin(t)$ with $y(0)=2$, $y'(0)=0$, and $y''(0)=0$

10. (10%) Solve the partial differential equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}+\sin(x)$ for $t>0$, $0<x<\pi$ with boundary condition $u(0,t)=0, u(\pi,t)=\pi$ and initial condition $u(x,0)=2\sin(x)$