※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (a) A particle, which is confined to move in one dimension between 0 and L, is described by the wave function

$$\psi(x) = Ax(L-x).$$

Use the normalization condition to determine the constant A and then derive an expression for the average value of the position of the particle. (10%)

(b) Use above result to derive an expression for the average value of the kinetic energy of the particle. (5%)

2. Show that the maximum kinetic energy E_k, called the Compton edge, that a recoiling electron can carry away from a Compton scattering event is given by

$$E_k = \frac{hf}{1+\frac{mC^2}{2hf}}$$

where f and C are the frequency and speed of incident photon, respectively, and h is Planck constant. (10%)

3. In this problem you are to obtain the Bohr results for the energy levels in hydrogen without using the quantization of angular momentum. In order to relate the total energy ($E_n = -ke^2/2r_n$) to the Balmer-Ritz formula, assume that the radii of allowed orbits are given by $r_n = n^2 r_0$, where n is an integer and r_0 is a constant to be determine.

(a) Show that the frequency of radiation for a transition to $n_f = n-1$ is given by $f \approx ke^2/hr_0n^3$ for large n. (5%)

(b) Show that the frequency of revolution is given by

$$f_{rev}^2 = \frac{ke^2}{4\pi^2 m r_0^3 n^6} \quad (5\%)$$

(c) Use the correspondence principle to determine r_0. (5%)

4. (a) Show that the wave function

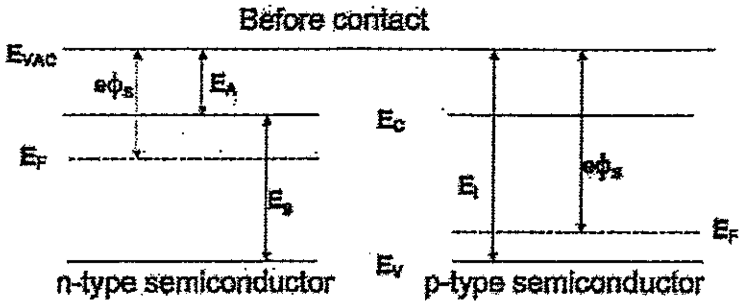
$$\psi(x) = Ae^{ikx}$$

represents a state for which the momentum of the particle has the value $p = \hbar k$. (5%)

(b) Find the kinetic energy of the particle in this state. (5%)

5. Semiconductor (10%)

Draw the band diagram when the semiconducting materials below are brought together to form homogeneous p-n junction in the condition of (a) under dark in equilibrium and (b) under light at open circuit.



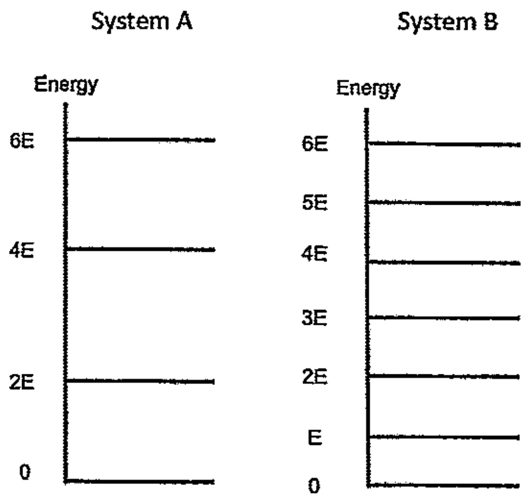
6. Stefan-Boltzmann and Wien displacement law (10%)

(a) Use Stefan–Boltzmann law to calculate the total power radiation per unit area by an object at a temperature of 3000K assuming the object is an ideal black body radiator. (5 points) [$\sigma = 5.7 \times 10^{-8}$ W/m^2K^4]

(b) Please describe Wien's displacement law. Assume that the sum radiates as a black body with a surface temperature of 5800K. Show the wavelength with maximum peak (in nm) in the solar spectrum. (5 points) [*Wien's displacement constant*=2.898×10^{-3} m K]

7. Statistics (15%)

Consider two systems having specific energy level given in the figure below. At equilibrium, suppose there are 4 particles to be distributed in the energy levels to receive total energy of 6E. Please (a) illustrate the available arrangements and the number of microstates available of each arrangement for both system if the particles follow Maxwell-Boltzmann distribution. (7 points) (b) Which arrangement is the most probable configuration for system B? And, find its probability of appearance? (3 points) (c) With the same energy of 6E for both systems, which one has higher entropy? (2 points) (d) If the particle is fermion, what is the number of available microstates for system B at the same equilibrium condition? (3 points)



8. Solid-state (15%)

The primitive lattice vectors in hexagonal can be described by :

$$\vec{a}_1 = \frac{\sqrt{3}a}{2}\vec{i} + \frac{a}{2}\vec{j} \qquad \vec{a}_2 = -\frac{\sqrt{3}a}{2}\vec{i} + \frac{a}{2}\vec{j} \qquad \vec{a}_3 = c\vec{k}.$$

where $\vec{i}$, $\vec{j}$ and $\vec{k}$ are the unit vectors of Cartesian coordinate system

Please

(a) Plot the structure of hexagonal closed packed (HCP) lattice. (2 points)

(b) Find the volume of the primitive cell. (3 points)

(c) Show the vectors of its reciprocal lattice. (6 points)

(d) Plot the first Brillouin zone of the hexagonal lattice. (4 points)