編號： 45

國立成功大學 108 學年度碩士班招生考試試題

系　　所：光電科學與工程學系

考試科目：工程數學

考試日期：0224，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. A cantilever beam is embedded at one end and free at the other end, Fig. 1. A diving board and an AFM cantilever are the examples of such beams. Assume $y(x)$ is the deflection of the cantilever beam. Please write down the boundary conditions at x=0 and x=L. (5%).

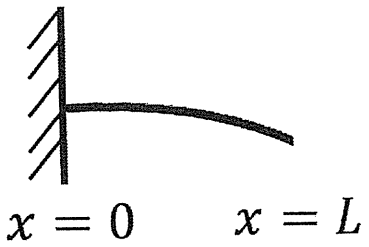

Fig. 1

2. Solve the boundary-value problem:

$y'' + ay = 0 \quad y(0) = 0, \; y(L) = 0.$

Please consider $a = 0$, $a < 0$, and $a > 0$ respectively. (5%, 5%, 15%).

3. Please use Fourier transform to solve the following one-dimension heat equation

$$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, -\infty < x < \infty, t > 0, and\ u(x,0) = g(x)\ , where\ g(x) = \begin{cases} u_0 & |x| < 1 \\ 0 & |x| > 1 \end{cases}$$

The Fourier transform of $u(x,t)$ is defined as $F\{u(x,t)\} = \int_{-\infty}^{\infty} u(x,t)e^{i\lambda x}dx \equiv U(\lambda,t)$.

First, solve $U(\lambda,t)$. (10%)

Second, solve and express $u(x,t)$ in terms of λ, t and x. (10%)

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4. Let $f = zy + yx$, $\vec{v} = [y,\ z,\ 4z - x]$, $\vec{w} = [y^2,\ z^2,\ x^2]$. Please find:

(a) $(\text{grad}\, f) \times \vec{v}$, (b) $\nabla^2 (f^2)$, (c) $\text{curl}\, \vec{w}$, (d) $D_{\vec{v}} f$ at (3, 7, 5). (5 %, 5%, 5 %, 5 %)

5. (a) Please evaluate the line integral $\int_C \vec{F}(\vec{r}) \cdot d\vec{r}$ with $\vec{F} = [9z, 5x, 3y]$ and *C* the ellipse

$x^2 + y^2 = 9$, $z = x + 2$. (5 %)

(b) Please evaluate the surface integral $\iint_S \vec{F} \cdot \vec{n}\, dA$ with $\vec{F} = [\sin^2 x,\ -y \sin 2x,\ 5z]$ and *S* the surface of the box $|x| \le a$, $|y| \le b$, $|z| \le c$. (5 %)

6. (a) Compute the function value of $\cosh(2n+1)\pi i$, n = 1, 2, ⋯. (5 %)

(b) Integrate $5z - 3/z$ counterclockwise around the unit circle. (5 %)

(c) Find the radius of convergence: $\sum_{n=1}^{\infty} \frac{n}{2^n} (z + i)^{2n}$. (5 %)

7. Find the linear fractional transformation that maps $z_1 = -2, z_2 = 0, z_3 = 2$ onto

$w_1 = \infty, w_2 = 1/4, w_3 = 3/8$, respectively. (5 %)