國立成功大學 108 學年度碩士班招生考試試題

系　所：光電科學與工程學系

考試科目：電磁學

考試日期：0224，節次：2

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (a) a point charge $\boldsymbol{q}$ is at a distance $\boldsymbol{d}$ above an infinite grounded conducting plate. Find the induced surface charge density σ and total charge q_{total} on the conducting plane. (5%)

   (b) Similar to (a), a point charge $\boldsymbol{q}$ is at distance $\boldsymbol{d}$ above a semi-infinite dielectric region (z<0) with dielectric constant $\varepsilon_r = 1+\chi_e$. Find the induced surface charge density σ_b and total charge $q_{b,total}$ at z=0 interface. (10%) (use image charge method, find V $\sigma = -\varepsilon_0 \left.\frac{\partial V}{\partial z}\right|_{z=0}$ $q_{total} = \int_{r=0}^{r=\infty}\int_{\phi=0}^{\phi=2\pi} \sigma r dr d\phi$

$$\sigma_b = \vec{P}\cdot\hat{n} = P_z = \varepsilon_0\chi_e E_z \quad E_z = E_{q,z} + E_b \quad E_{q,z} = -E_q\cos\theta \quad E_b = -\frac{\sigma_b}{2\varepsilon_0}$$

2. (a) An uncharged metal sphere of radius R is placed in a uniform electric field $\vec{E} = E_0\hat{z}$. Find the *potential* V *outside* the sphere and the *induced surface charge density* $\sigma = -\varepsilon_0 \left.\frac{\partial V}{\partial r}\right|_{r=R}$ (8%)

   (b) A *dielectric sphere* of homogenous linear material with relative dielectric constant $\varepsilon_r = 1+\chi_e$ is placed in an uniform electric field $\vec{E} = E_0\hat{z}$. Find the *potential* V and *electric field* E *inside* the sphere. (7%)

(use separation of variable in spherical Coordinate. $V(r,\theta) = \sum_{l=0}^{\infty}(A_l r^l + B_l \frac{1}{r^{l+1}})P_l(\cos\theta)$ with Legendre Polynomial $P_0(\cos\theta) = 1, P_1(\cos\theta) = \cos\theta,\ P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1), \int_{-1}^{1} P_l(x)P_{l'}(x)dx = \int_0^{\pi} P_l(\cos\theta)P_{l'}(\cos\theta)\sin\theta d\theta = \begin{cases} 0 & l\neq l' \\ \frac{2}{2l+1} & l = l' \end{cases}$

(a) B.C (i) $V=0$ at $r=R$ (ii) $V = -E_0 r\cos\theta$, $r>>R$, $\nabla V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\hat{\phi}$

(b) B.C. (i) $V_{in}=V_{out}$ at $r=R$ (ii) $\varepsilon_0\frac{\partial V_{out}}{\partial r} = \varepsilon_0\varepsilon_r\frac{\partial V_{in}}{\partial r}$, $r=R$ (iii) $V = -E_0 r\cos\theta$, $r>>R$, $\nabla V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}$,(or you can use iteration method)

3. A infinite-thin metal with sheet conductivity $\sigma(x) = \sigma_0\delta(x)$ is located at $x=0$. For a *normal* incident plane wave propagating in $\boldsymbol{x}$ direction with $\boldsymbol{E}$ field polarized in $\boldsymbol{y}$ direction, find the transmission coefficient $\boldsymbol{t}$ and reflection coefficient $\boldsymbol{r}$=? (use methods similar to the derivation of the transmission/reflection coefficient of normal incident to an dielectric interface, except now there is a sheet current density $J_y(x) = \sigma_y(x)E_y(x) = \sigma_0\delta(x)E_y(x)$ in $j\omega\varepsilon_0 E_y + \sigma_0\delta(x)E_y = (\nabla\times H)_y = -\frac{\partial H_z}{\partial x}$ (10%)

4. Similar to previous problem, now place a perfect electric conductor (PEC) wall a distance $\boldsymbol{d}$ after the infinite-thin metal. A plane wave in normal incident toward the metal and the PEC wall. Under what condition (d=? and r=?), there will be *no energy reflected* (thus the incident wave is total absorbed by the metal). (Use Fabry-Perot ray-optics method to derive the total reflection coefficient Ar =Ar$_1$+Ar$_2$+Ar$_3$+ .... of the combined metal sheet (r_1,t_1) and PEC wall (r_2,t_2) (10%)

5. Ionized gases with equal electron and ion densities are called plasmas. If, in the plasma, the motion of the ions and the collision between the electrons and the gas atoms and molecules are ignored.

(a) Prove that the equivalent permittivity of the plasma with an electron density N

$$\varepsilon = \varepsilon_0 \left(1 - \frac{\omega_p^2}{\omega^2}\right) \quad (F/m),$$

where $\omega_p = \sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$ (rad/s) is the plasma frequency, ω is the angular frequency of electromagnetic waves, m is the electron mass and ε_0 is the permittivity of free space.

(Hint: You can regard the plasma as a free electron gas and assume a time-harmonic electric field E in the x-direction at angular frequency ω, and then write down the force equation for an electron with charge $-e$ and mass m.) (15%)

(b) Discuss the wave propagation for the cases of $\omega > \omega_p$ and $\omega < \omega_p$, respectively. (5%)

6. You are fishing near a large lake in the afternoon. The reflection from the sun makes it hard for you to look into the water. You have a linear polarizer. What can you do? Explain the physics behind your answer. Please describe it in terms of the orientation of the transmission axis. (10%)

7. Consider $\omega=2\pi c/\lambda_0$ and $k=2\pi n/\lambda_0$. In a dispersive medium, find that the expression of the group velocity is as follows: (10%)

$$V_g = \frac{c}{n - \lambda_0 (\frac{dn}{d\lambda_0})}$$

8. A hollow rectangular waveguide is used to transmit signals at a carrier frequency of 6 GHz. Design the waveguide so that the cut-off frequency of the dominant TE mode is lower than the carrier by 25% and the next mode is at least 25% higher than the carrier frequency? (10%)