國立成功大學 109 學年度碩士班招生考試試題

系　所：光電科學與工程學系

考試科目：工程數學

考試日期：0211，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (a) Show that the given matrix **A** is diagonalizable. **A**= $\begin{pmatrix} -8 & -10 & 7 & -9 \\ 0 & 2 & 0 & 0 \\ -9 & -9 & 8 & -9 \\ 1 & 1 & -1 & 2 \end{pmatrix}$

   (b) Find the matrix **P** that diagonalizes **A** and the diagonal matrix **D** such that **D**= **P**$^{-1}$**AP**.

   (c) Find the 10th power of **A**. (5%, 5%, 10%)

2. (a) Show that $\int_c \vec{F} \cdot d\vec{r}$, where $\vec{F}(x,y) = (y^2 - 6xy + 6)\vec{i} + (2xy - 3x^2 - 2y)\vec{j}$, is independent of the path C between (-1,0) and (3,4).

   (b) Find a potential function φ for $\vec{F}$.

   (c) Evaluate $\int_{(-1,0)}^{(3,4)} \vec{F} \cdot d\vec{r}$. (5%, 5%, 5%)

3. Solve the boundary value problem. (5%)

   $$x^2y'' - 4xy' + 6y = x^4, y(1) - y'(1) = 0, y(3) = 0$$

4. Expand $f(x) = x^4, -1 < x < 1$, in a Fourier series. (5%)

5. Solve the given initial value problem (5%)

   $$\frac{d^2x}{dt^2} + 9x = 5sin3t,\ x(0) = 2, x'(0) = 0.$$

6. Solve Laplace's equation,

   $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,$$

   with the boundary conditions: $u(x, 0)=0$, $u(x, 1)=0$, $u(0, y)=F(y)$, $u(1, y)=0$

   where $F(y) = y, \quad 0 \le y \le 1/2$
   $F(y) = 1 - y, \quad 1/2 \le y \le 1$ (20%)

7. Suppose $f(z)$ is analytic on the closed disk of radius r centered at z_0, prove that

$$f(z_0) = \frac{1}{2\pi}\int_0^{2\pi} f(z_0 + re^{i\theta})d\theta$$

(10%)

8. Prove that

$$\int_{-\infty}^{\infty} f(t)g^*(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)G^*(\omega)d\omega$$

where $F(\omega)$ and $G(\omega)$ are the Fourier transforms of $f(t)$ and $g(t)$, respectively. (10%)

Hint: $\delta(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{\pm i\tau\omega}d\tau$

9. Find the first four terms of the Taylor series about z=3 of the function

$$f(z) = \frac{1}{5-z}$$

(10%)