

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Useful constants

Planck constant $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$; $e^2/4\pi\epsilon_0 = 1.44 \text{ MeV}\cdot\text{fm}$; Bohr radius = 0.053 nm;

Rest energy of electron $m_e c^2 = 0.5 \text{ MeV}$

- Radiation of wavelength $\lambda = 290 \text{ nm}$ falls on a metal surface for which the work function is $W = 4.05 \text{ eV}$. What potential is needed to stop the most energetic photoelectrons? (10%)
- Consider a wave packet for which

$$A(k) = \begin{cases} N & -K \leq k \leq K \\ 0 & \text{elsewhere} \end{cases}$$

- Calculate $\psi(x, 0)$. (5%)
 - Use a reasonable definition of the width to show that $\Delta k \Delta x > \frac{1}{2}$ is satisfied. (5%)
- Consider a particle whose normalized wave function is

$$\psi(x) = \begin{cases} 2\alpha\sqrt{\alpha}xe^{-\alpha x} & x > 0 \\ 0 & x < 0 \end{cases}$$

- For what value of x does $P(x) = |\psi(x)|^2$ peak? (5%)
 - Calculate $\langle x \rangle$ and $\langle x^2 \rangle$. (5%)
 - What is the probability that the particle is found between $x = 0$ and $x = 1/\alpha$? (5%)
 - Calculate $\phi(p)$ and use this to calculate $\langle p \rangle$ and $\langle p^2 \rangle$. (5%)
- The wave function for a particle is given by

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

What flux does this represent? (10%)

- In an electron microscope we wish to study particles of diameter about $0.10 \mu\text{m}$.
 - What should be the de Broglie wavelength of the electrons? (5%)
 - What potential difference should the electrons be accelerated to have that de Broglie wavelength? (5%)
- X-ray photons of wavelength 0.02480 nm are incident on a target and the Compton-scattered photons are observed at an angle of 90.0 degrees.
 - What is the momentum of the scattered photons? (5%)
 - What is the kinetic energy of the scattered electrons? (5%)
- In the Rutherford scattering, an α -particle is elastic scattered by a gold nucleus in to angle.
 - Derive and show the momentum transfer p is: $\Delta p = 2p \sin \frac{\theta}{2}$ (5%)
 - Calculate the kinetic energy of an α -particle ($Z=2$) if the distance of closest approach to a gold nucleus ($Z=79$) is 10 fm , when scattered at 90 degrees. (5%)
- In the Bohr model of the hydrogen atom, an electron transition occurs from $n = 3$ to $n=2$.

- a) What is the energy of the emitted radiation? (4%)
 - b) Compare (a) to the orbital frequency of an electron with $n = 3$ to $n=2$ that it can radiate. (6%)
9. Consider the normal Zeeman effect applied to the $3d$ to $2p$ transition.
- a) Sketch an energy level diagram that shows the splitting of the $3d$ and $2p$ levels in an external magnetic field. Indicate all possible transitions from each m_l state of the $3d$ level to each m_l state of the $2p$ level. (5%)
 - b) How many different transition energies can be emitted? Explain it. (5%)