編號： 46

國立成功大學 109 學年度碩士班招生考試試題

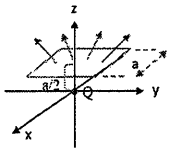
系　　所：光電科學與工程學系

考試科目：電磁學　　考試日期：0211，節次：2

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) (a)E is a vector, prove $\nabla\times\left(\nabla\times\vec{E}\right)=\nabla\left(\nabla\cdot\vec{E}\right)-\nabla^2\vec{E}$ $\left(\nabla=\frac{\partial}{\partial x}\hat{x}+\frac{\partial}{\partial y}\hat{y}+\frac{\partial}{\partial z}\hat{z}\right)$

(b) For a point charge Q located at the origin, find the electric flux through the square surface (side a) located right above the charge with distance a/2



2. (15%) A sphere with homogeneous linear **dielectric** material $\varepsilon_r=1+\chi_e$ is placed in a uniform electric field $\vec{E}=E_0\hat{z}$. (a) find the potential inside V(r<R) and outside V(r>R) the sphere by using the separation of variables method

$V(r,\theta)=\sum_{l=0}^{\infty}(A_l r^l+B_l\frac{1}{r^{l+1}})P_l(cos\theta)\quad r\geq R$

B.C. (i) $V_{in}=V_{out}$ *at* r=R (ii) $\varepsilon\frac{\partial V_{in}}{\partial r}=\varepsilon_0\frac{\partial V_{out}}{\partial r}$ *at* $r=R$ (iii) $V_{out}\to -E_0 r\cos\theta \quad r\gg R$, $z=r\cos\theta$

(b) Find the electric field inside E_{in} (r<R) and outside E_{out}(r>R) $\left(\vec{E}=-\nabla V=-\frac{\partial V}{\partial r}\hat{r}-\frac{1}{r}\frac{\partial V}{\partial\theta}\hat{\theta}-\frac{1}{r\sin\theta}\frac{\partial V}{\partial\theta}\hat{\phi}\right)$

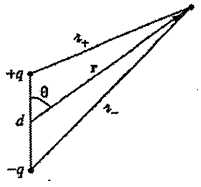
(c) Find the Polarization Density **P** inside the dielectric sphere and the induced Surface Charge Density σ_b

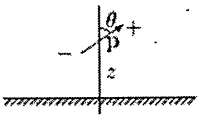
$\left(P_{in}=\varepsilon_0\chi_e E_{in},\sigma_b=P\cdot\hat{n}\right)$

3. (10%)(a)A electric dipole **p**(=qd) consists of two equal and opposite charges ±**q** separated by a distance **d** pointing at the **z** direction. At a point (r, θ , ϕ) far from the dipole (r>>d), find the approximated potential V and electric field E

$\left(V=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_+}-\frac{q}{r_-}\right),\ r_\pm=r\left(1+\left(\frac{d}{2r}\right)^2\mp\frac{d}{r}\cos\theta\right)^{\frac{1}{2}}\ ,\left(\frac{1}{1\pm x}\right)^k\approx 1\mp kx+\ldots\ldots,\ \vec{E}=-\nabla V\right)$

(b) A dipole is at distance z above a grounded metal plane. Find the torque $N=p\times E$ on the dipole and the rotation direction of the dipole if $\theta<\frac{\pi}{2}$. (hint: use the image method of the dipole.)





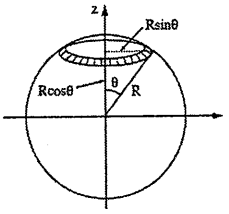
4. (15%) A spherical shell with surface charge density σ and radius **R** spinning with constant angular velocity ω

(a) Find the magnetic dipole moment **m** =IA of the spinning spherical shell.

(b) Find the vector potential **A** for r>>R by using the dipole approximation. $\vec{A}_{dipole}(\vec{r})=\frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$

(c) Find the magnetic field **B** outside the spherical shell at (r, θ , ϕ) for r>R

$\vec{B}=\nabla\times\vec{A}=\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\left(\sin\theta A_\phi\right)-\frac{\partial A_\theta}{\partial\phi}\right)\hat{r}+\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi}-\frac{\partial}{\partial r}\left(rA_\phi\right)\right)\hat{\theta}+\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\theta\right)-\frac{\partial A_r}{\partial\theta}\right]\hat{\phi}$



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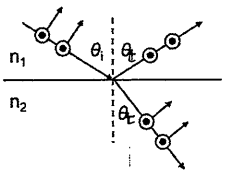
5. (10%) (a) Please write down the Maxwell equations (4 equations) with free charge ρ_f and free current density $\vec{J}_f$, and describe their physical significant.

(b) Please derive four boundary conditions of electromagnetic fields based on the Maxwell equations.

6. (10%) (a) It is known that the reflection intensity is zero when a TM wave is obliquely incident from an optically thinner medium (refractive index n_1) onto an optically thicker medium (refractive index n_2) with Brewster's angle θ_B. Please derive that the Brewster's angle $\theta_B = \tan^{-1}(\frac{n_2}{n_1})$.

(b) Following the previous question, when an un-polarized light illuminates onto the medium interface with Brewster's angle (that is, $\theta_i = \theta_B$), the reflected light will become a TE wave (see the figure on the right). Please derive that the $\theta_t = 90° - \theta_B$ under such condition.



7. (15%) An electromagnetic wave propagates in air, in which the instantaneous expression of the electric field is given by $\vec{E}(x,t) = \hat{a}_z 60\pi \cos(6\pi \times 10^6 t + \beta x)$ (V/m), please find (a) the direction of propagation and frequency f (b) the phase constant β and wavelength λ (c) the instantaneous expression of the magnetic field.

8. (15%) For an dielectric-filled (ε_r = 2.25) parallel-plate waveguide in which the distance between two plates a = 5 cm:

(a) Please find the cutoff frequency of TEM, TM_1, TE_1, TM_2, and TE_2 modes.

(b) If now an electromagnetic signal with frequency f = 3 GHz is considered, which modes can be excited?

(c) Following the previous question, please respectively determine the phase and group velocities (in term of light speed in vacuum c) of those guide modes which can be excited at a frequency of 3 GHz.